Shell model

Summary for the PhD lecture on Nuclear Physics

SHELL MODEL

Basic ideas

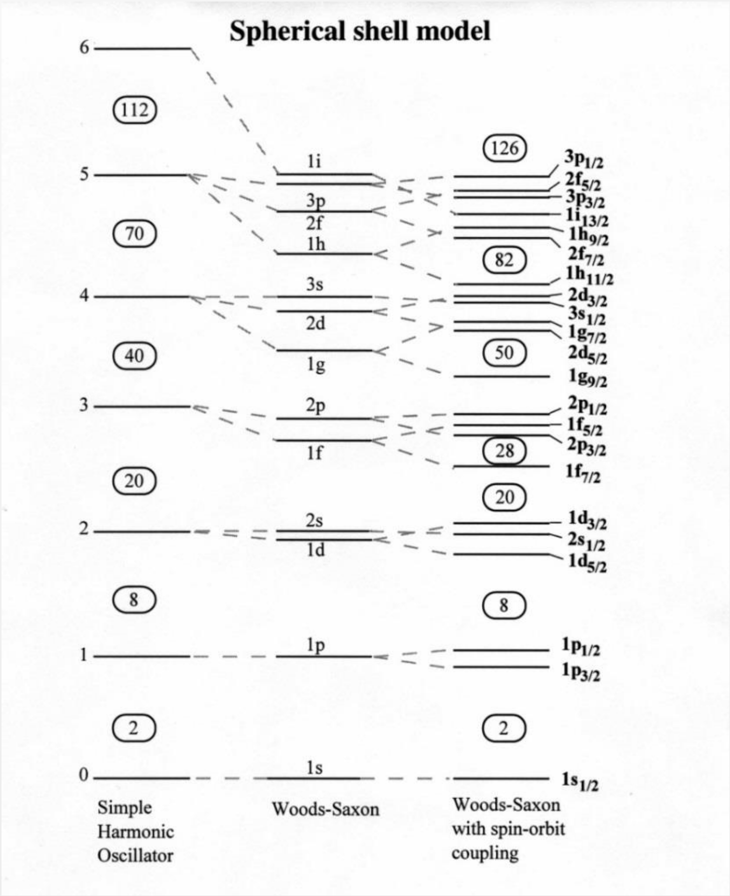
- Shell model explains several anomalies known for nuclei with “magic” number of proton and/or neutrons 2, 8, 20, 28, 50, 126 e.g. high-lying excited states for some isotopes, high separation energies, high abundance in solar system, resonant nature of neutron capture
- Basic principle is the single-particle shell model.
- Instead of detail description by nucleon-nucleon interaction we introduce the potential generated by the nuclear forces. Within this potential are localized orbital occupied by nucleons.
- In contrast to the electrons In atomic shell we don’t have central potential here, but rather uniformly distributed potential (well, not exactly, especially at the border – see later)

Shell occupation

Spectroscopic notation

ℓ **value**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Symbol	s	p	d	f	g	h	i

Note:
Number at the first position of level notation (i.e. **1**$d_{3/2}$ **3**$p_{3/2}$ **1**$h_{9/2}$...) don't have the role of principal quantum numbers (as in the case of atomic orbitals). Their purpose is only to note the order of levels with relevant *orbital angular momentum quantum number*

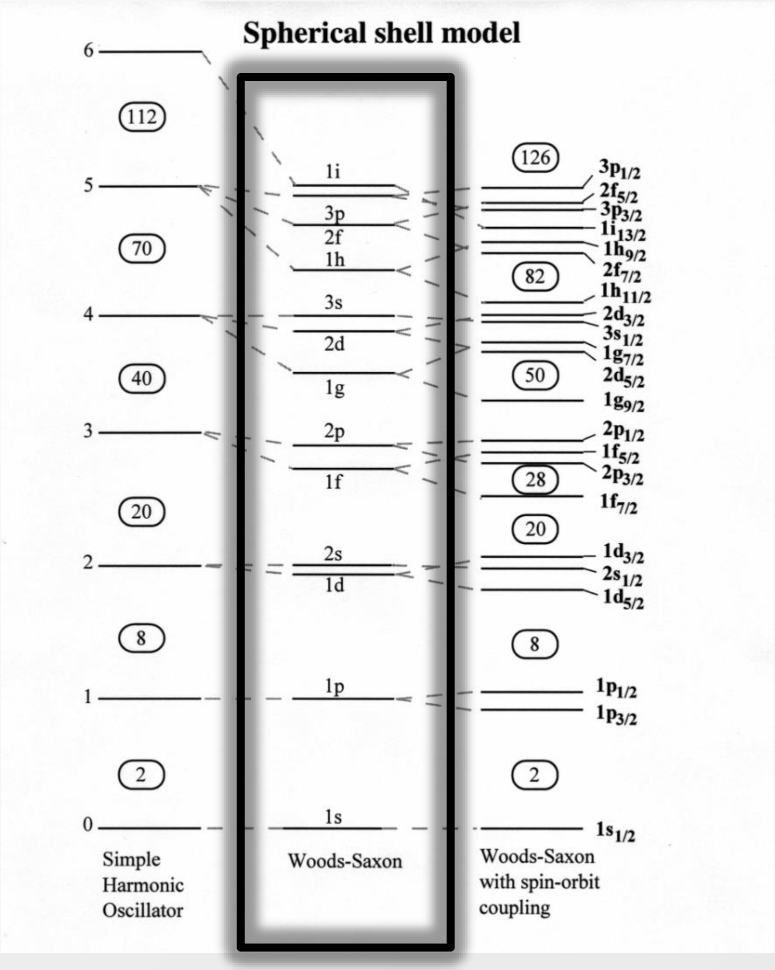


Number of nucleons at the level?

Number of particles at the level for realistic potential (Woods-Saxon) w/o spin-orbital interaction is 2 (2ℓ+1)

Factor (2ℓ+1) is from degeneracy of angular momentum and could have the value from – ℓ to + ℓ

Factor 2 is from degeneracy according to intristic spin of nucleon m_s

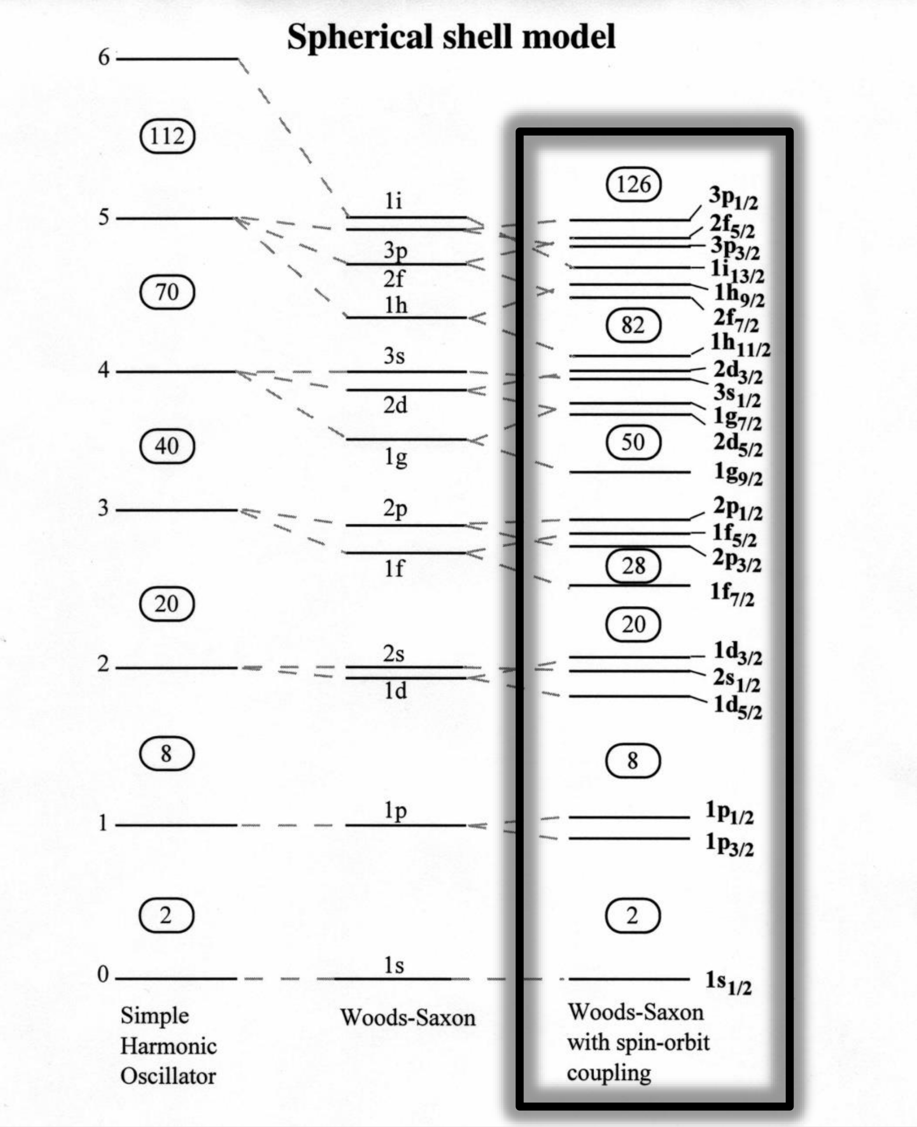


Spin-orbital interaction

Correction introduced by Meyer, Haxel, Suess and Jensen in 1940

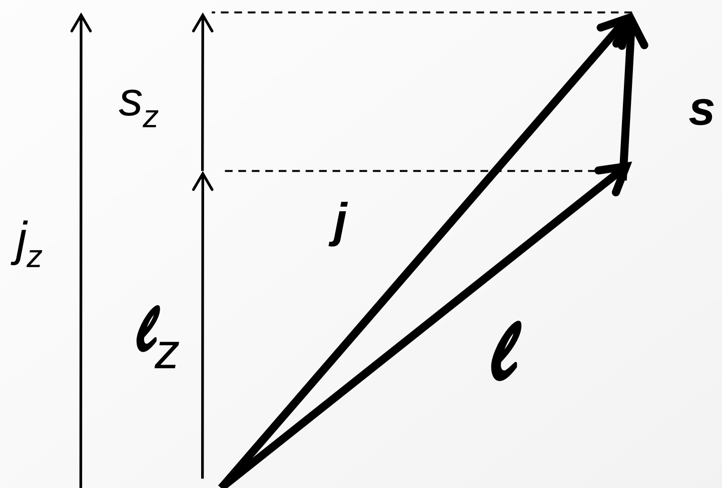
Idea is borrowed from the atomic physics, where we know the fine-structure of spectral lines due to the interaction of magnetic moment of electron with magnetic field generated by its movement in the vicinity of atomic nucleus. This effect itself is relatively weak 1:10^5.

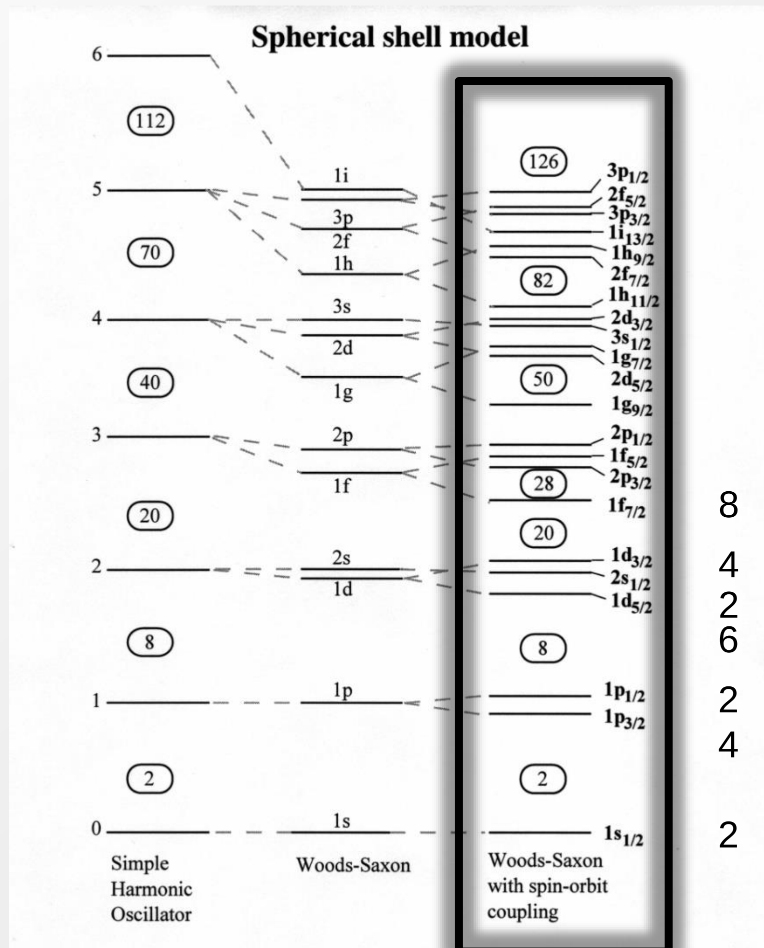
We took this concept, although we couldn't refer to the elmag interaction, which is too weak to explain the splitting of levels.



Realistic description with introduced ℓ•s interaction

For more realistic potential (Woods-Saxon) in combination with spin-orbital interaction we define total angular momentum $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$





8
4
2
6
2
4
2

The parity correspond to the value $(-1)^l$ indicated by the upper right index.

The parity ... Representation of the symmetry according to change of coordinates. For spherical coordinates term $(-1)^l$ in angular term

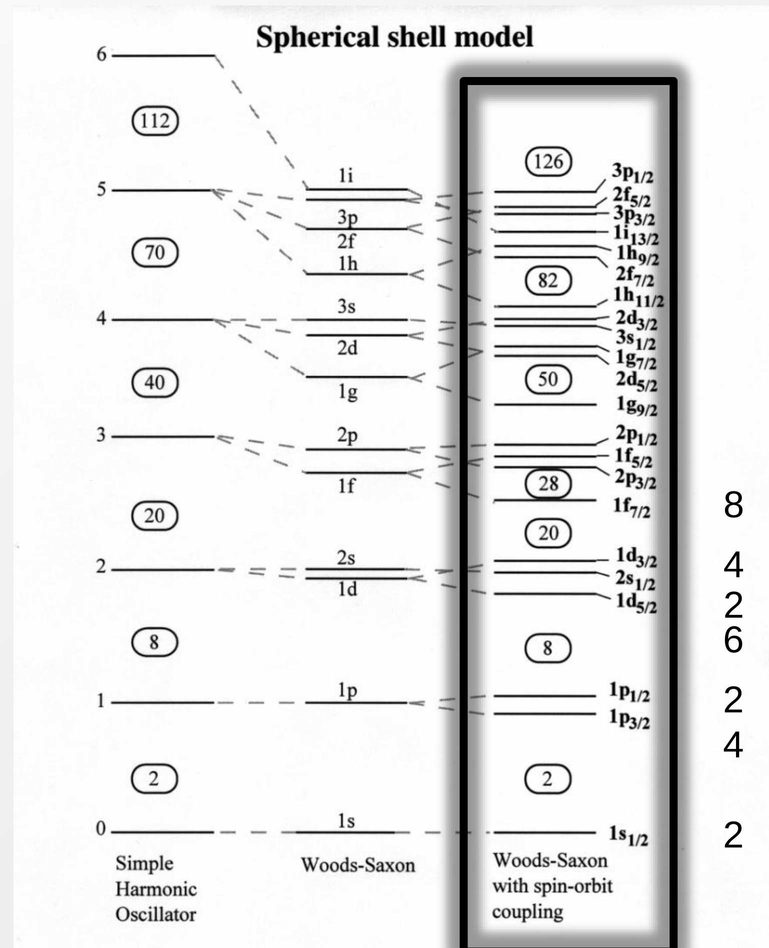
How many nucleons do we have at each level?

For more realistic potential (Woods-Saxon) in combination with spin-orbital interaction we define total angular momentum $\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$

Possible $m_j = m_\ell + m_s = m_\ell \pm ½$
Since m_ℓ is the integer, the m_j is always „half-number“ (± 1/2 ± 3/2 ± 5/2 ...)

Resulting total angular momentum ***j*** could be either $\boldsymbol{\ell}$ **+** ½ or $\boldsymbol{\ell}$ **-** ½ (except of $\ell = 0$ for which only ½ is possible) is indicated in subscript. (note, there are two possibilities since for two vectors we could get all possible combination).

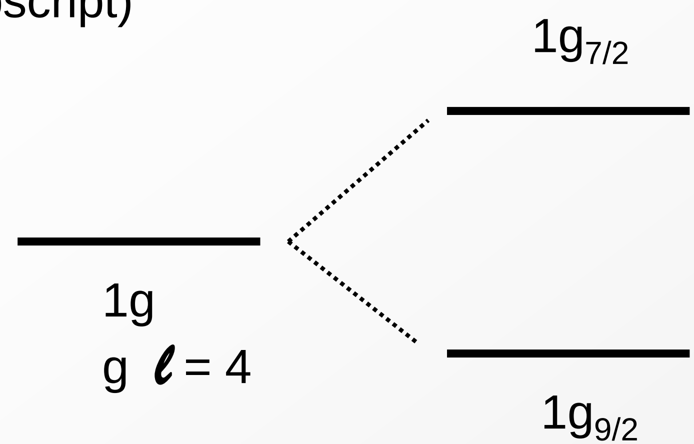
The corresponding number of nucleons is (2j+1)... e.g. for $1p_{3/2}$ we could have possible vlaues 3/2, 1/2,-1/2, -3/2



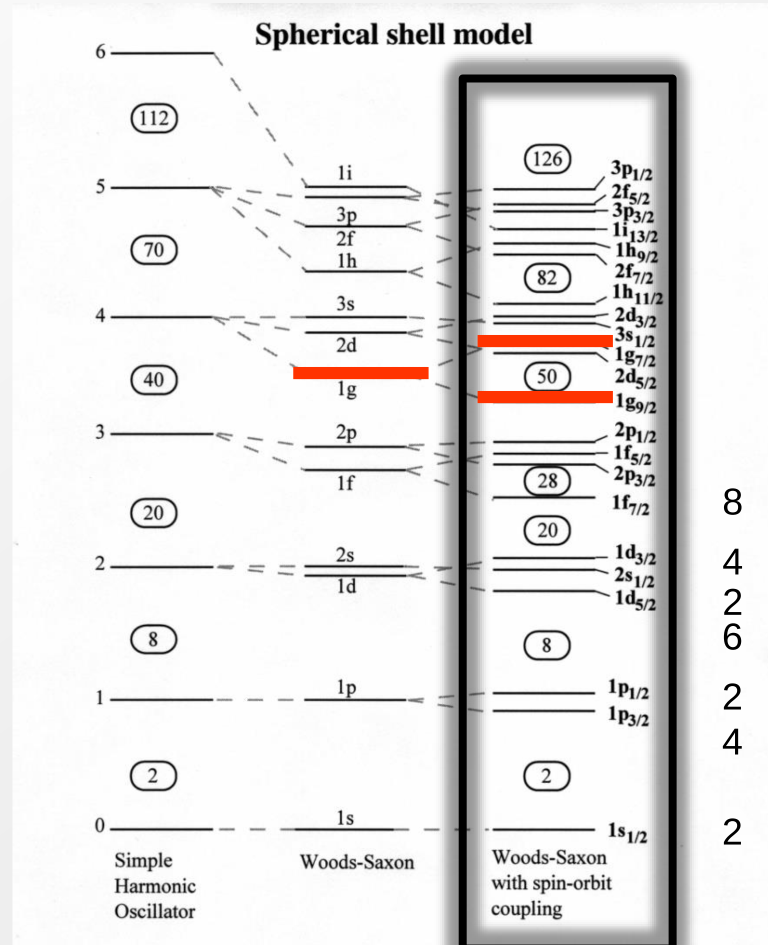
8
4
2
6
2
4
2

Level split

After combination of the orbital angular momentum and intristic spin the re are possible values for ***j*** either ***ℓ*** **+** ½ or ***ℓ*** **-** ½ (see right subscript)



The level, wiht higher total quantum number is pushed lower in energy considering negative $V_{so}(r)$.



8
4
2
6
2
4
2

„Energy difference“ of ℓ.s interaction

We introduced total angular momentum

$\boldsymbol{j} = \boldsymbol{\ell} + \boldsymbol{s}$

Possible value for total angular momentum $\boldsymbol{j}$ are:

$j = \ell + 1/2$ a $j = \ell - ½$

The energy correspond to mean value $< \boldsymbol{\ell} \bullet \boldsymbol{s} >$ obtained by classical trick:

$j^2 = (\ell + s)^2 = \ell^2 + 2\ell \bullet s + s^2$

Afterwards:

$\ell \bullet s = ½\,(j^2 - \ell^2 - s^2)$

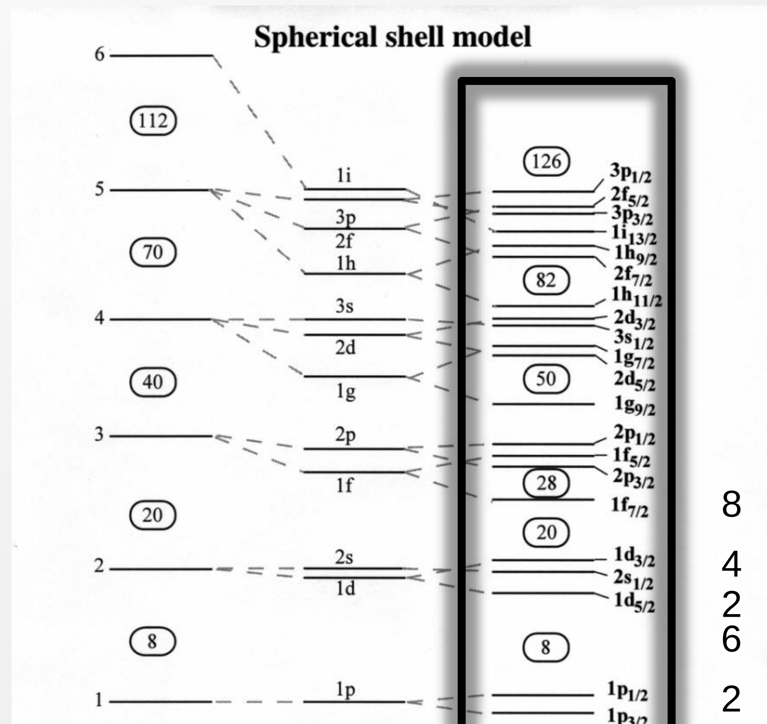
And mean value of *l.s* interaction is

$< \ell \bullet s> = ½\,[j(j+1) - \ell(\ell+1) - s(s+1)]\,\hbar^2$

The energy difference for spin-orbital pais with

$j = \ell + 1/2$ a $j = \ell - ½$ is afterwards:

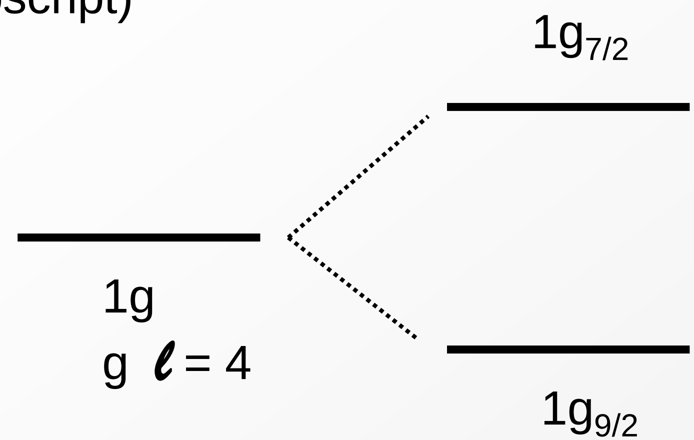
$<\ell \bullet s>_{\ell + 1/2} - <\ell \bullet s>_{\ell - 1/2} = ½\,(2\,\ell + 1)\,\hbar^2$



The energy difference increases with ℓ. Thus e.g. i orbital spilitting would be more significant compate to p orbital

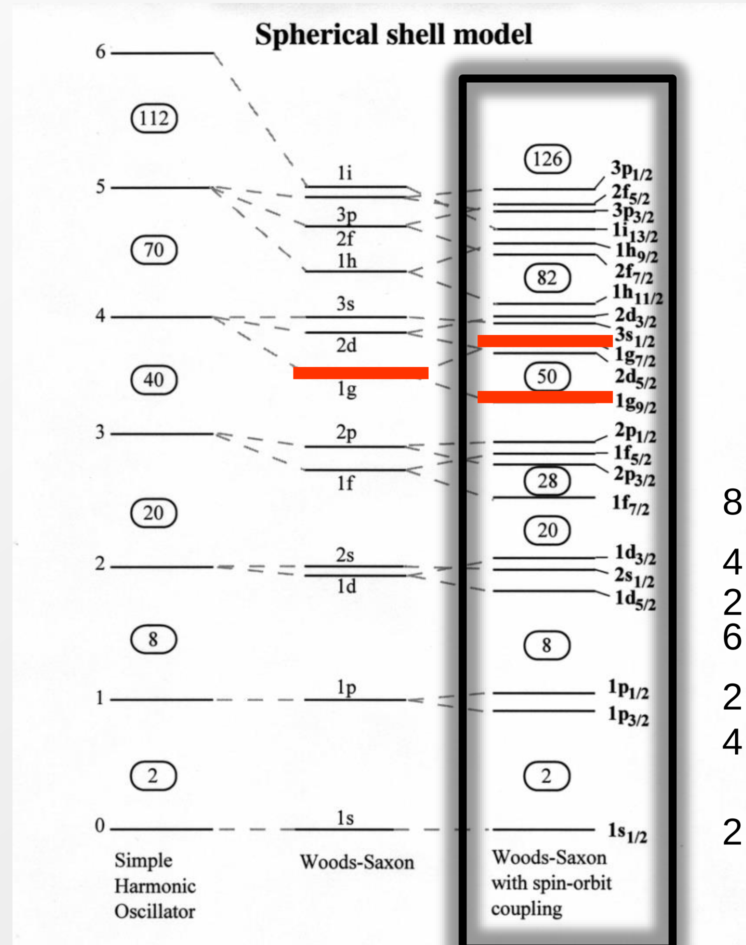
Level split

After combination of the orbital angular momentum and intristic spin the re are possible values for ***j*** either ℓ + ½ or ℓ - ½ (see right subscript)



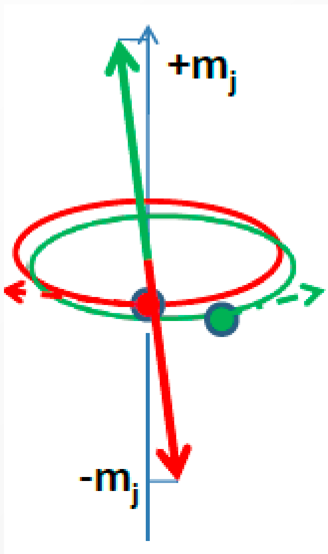
The level, wiht higher total quantum number is pushed lower in energy considering negative $V_{so}(r)$.

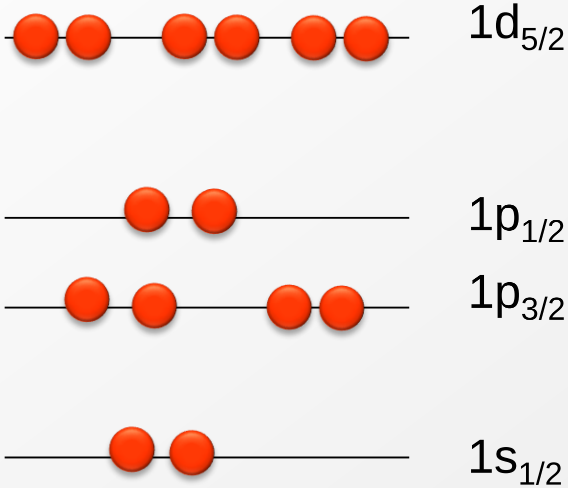
This split and shift is critical for high density of leves in case of heavy nuclei (e.g. hladiny $1g_{7/2}$ a $2d_{5/2}$ above closed shell of 50 nucleons) and might lead to possible incosistencies between theory and experiment



Nucleon pairing

- Short-distance nucleon-nucleon interaction combines nucleons into the pairs at time-reversed orbitals with oposite values of m_j. Therefore, all even-even nuclei have the spin and parity of their ground-state 0+.
- As an consequence we have the pairing term in the semi-empirical term for binding energy (see Bethe Wezsäcker formula)

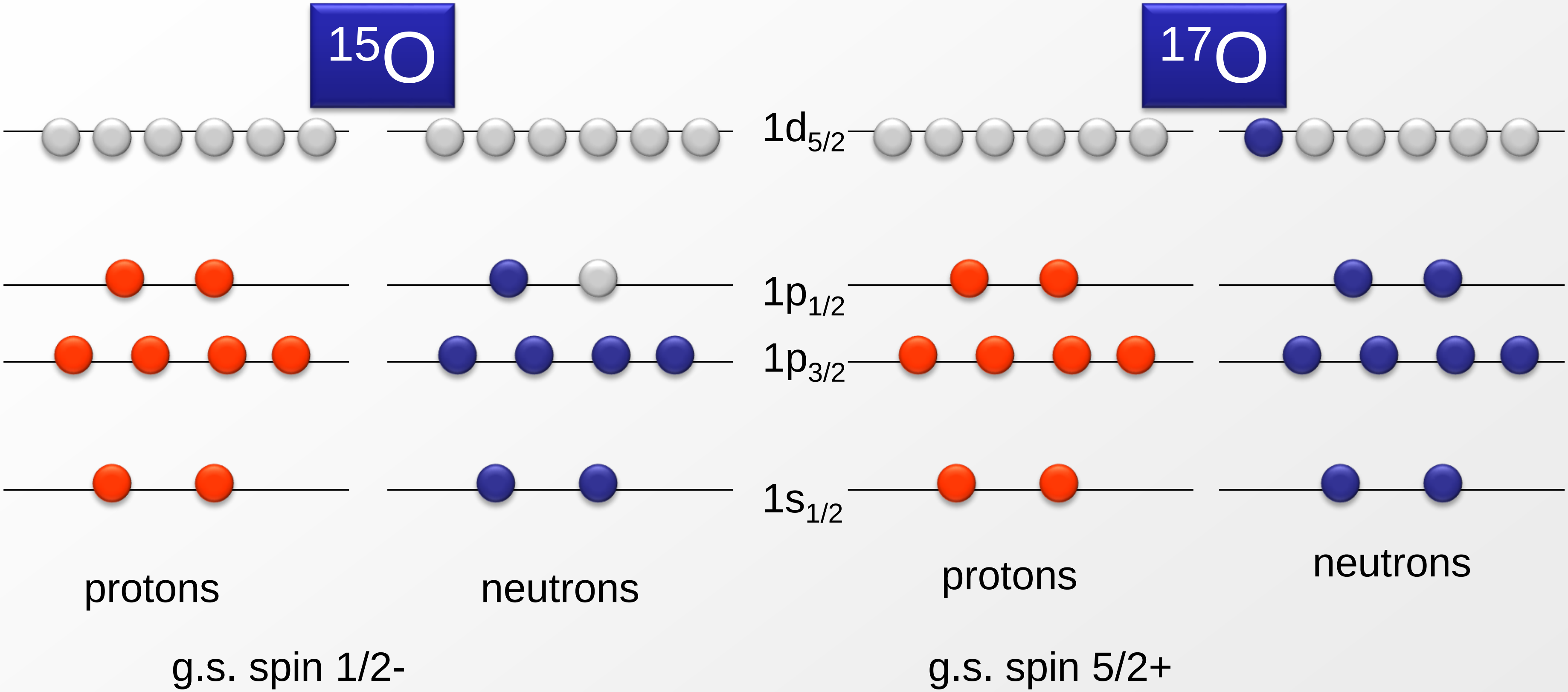




Aage N. Bohr, Ben. R. Motellson a Leo J. Rainwater

Nobelo prize in 1975 - „*for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection*"

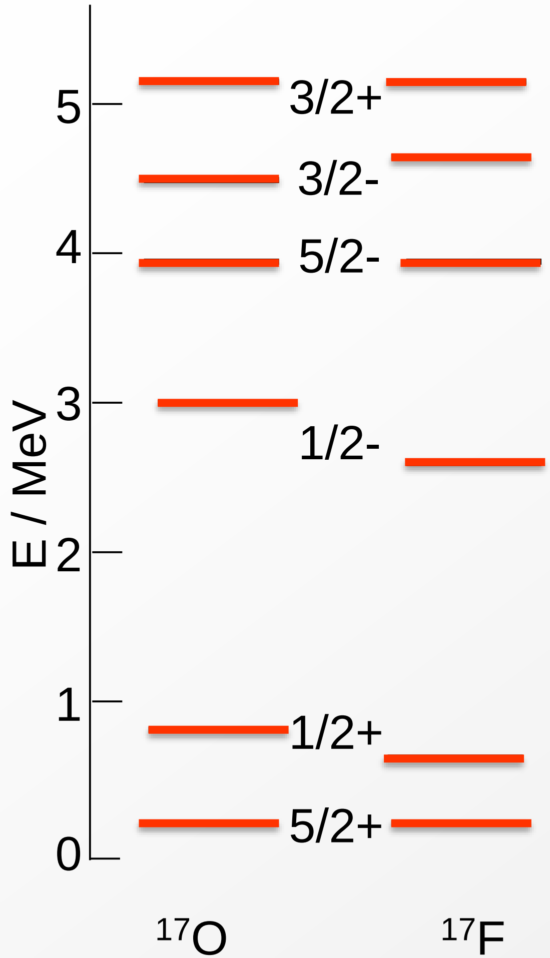
Example for the level occupation of ^{15}O vs. ^{17}O

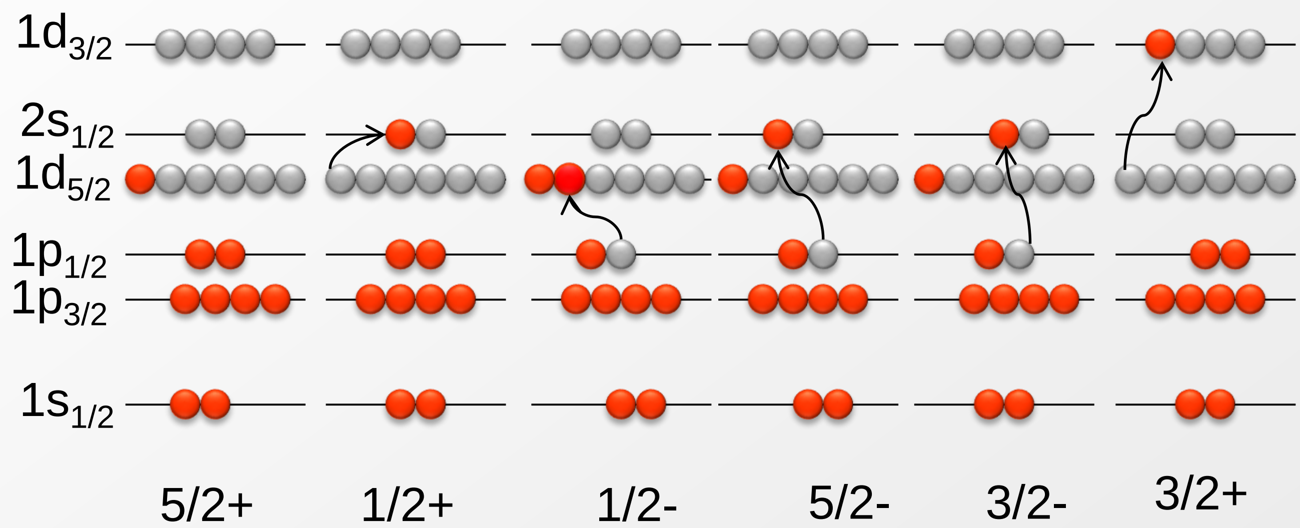


Unpaired nucleon defines the spin and parity for whole nucleus

Excitations in the framework of basic shell model

Comparison of ^{17}O (unpaired neutron) and ^{17}F (unpaired proton)





Odd-odd nuclei

What to do in the case of unpaired proton and neutron in odd-odd nuclei. There are available so called Brennan-Bernstein rules.

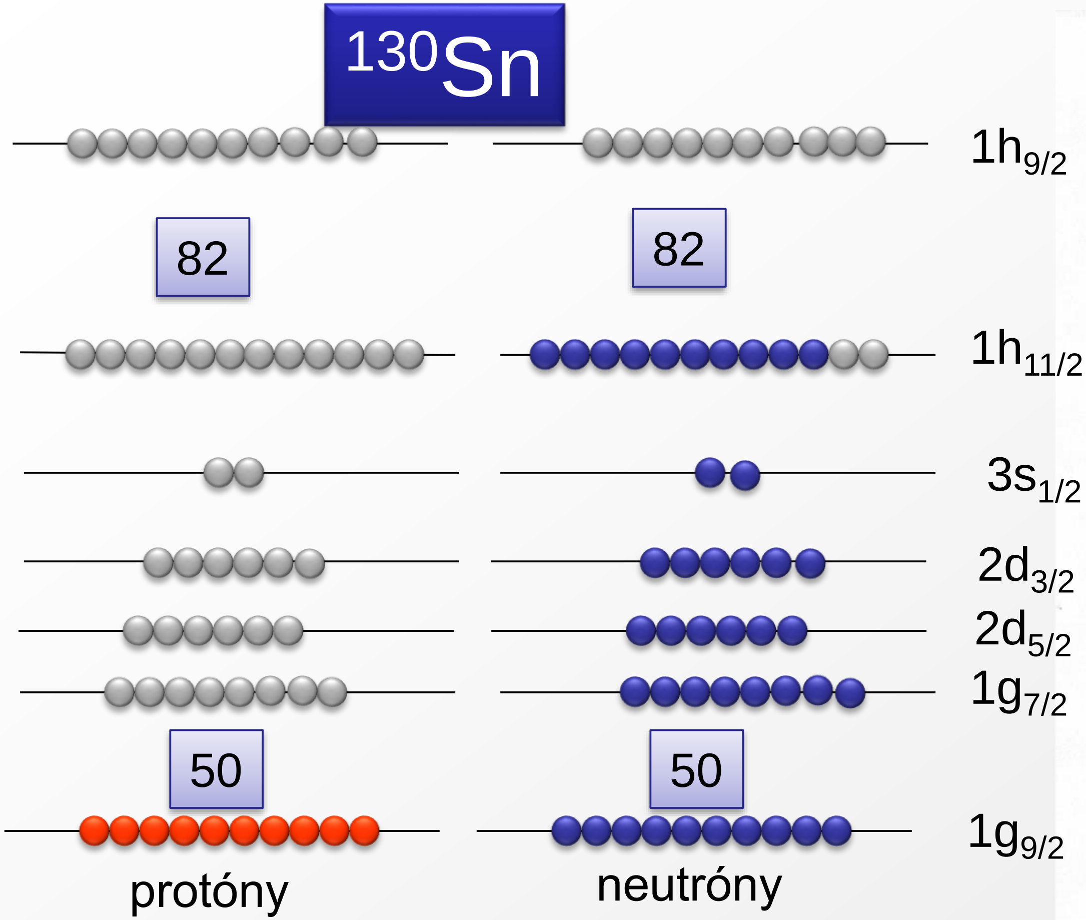
I. For particles of the same kind (either holes or particles) :
   a. $j_1 = \ell_1 \pm ½$ and $j_2 = \ell_2 \mp ½$, then $I = |j_1 - j_2|$
   b. $j_1 = \ell_1 \pm ½$ a $j_2 = \ell_2 \pm ½$, then $I = |j_1 \pm j_2|$

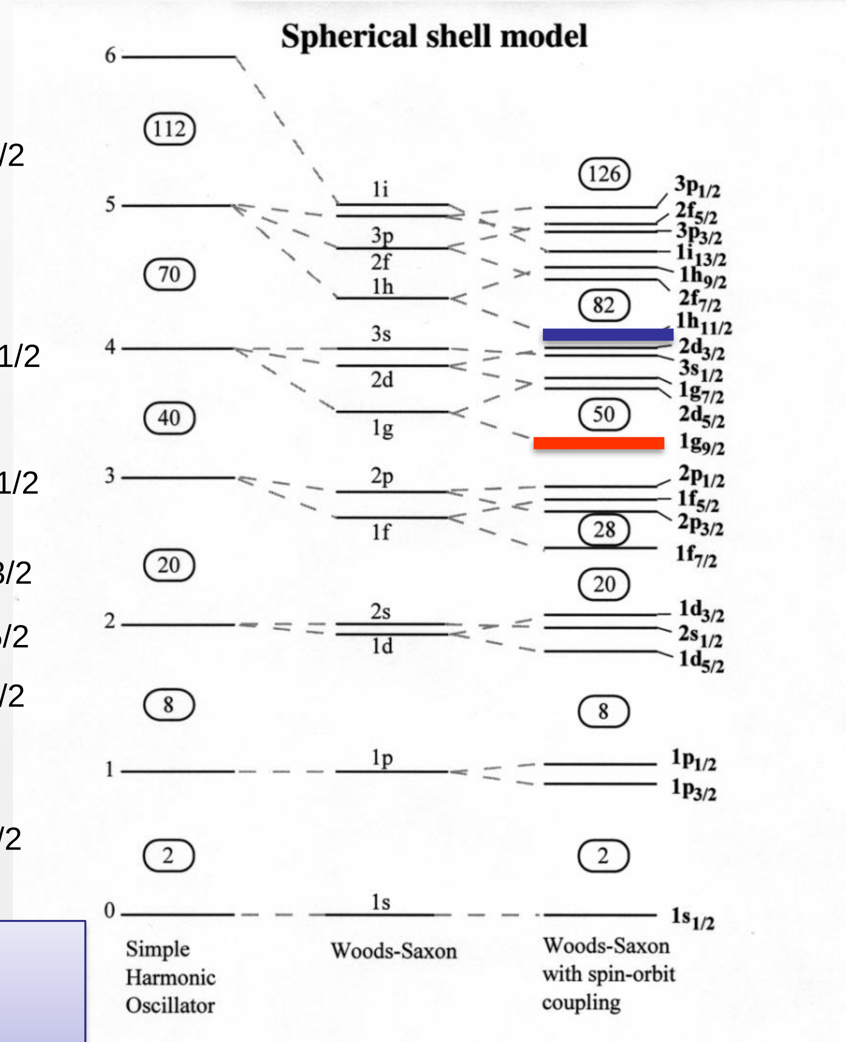
II. For particle-hole combination $I = j_1 + j_2 - 1$

Example: What is total angular momentum for ^{32}P?

Solution: 15 protons (last at $2s_{1/2}$) and 17 neutrons (last at $1d_{3/2}$). The total angular momentum is either 1+ or 2+.
For ^{32}P we have orbital and intristic spin for proton parallel ↑↑ and for neutron antiparallel ↑↓. Therofe the total angular momentum is
$I = |j_1 - j_2| = 1$. Considering parities is the result 1+ (exp. value 1+)

Even-even nuclei



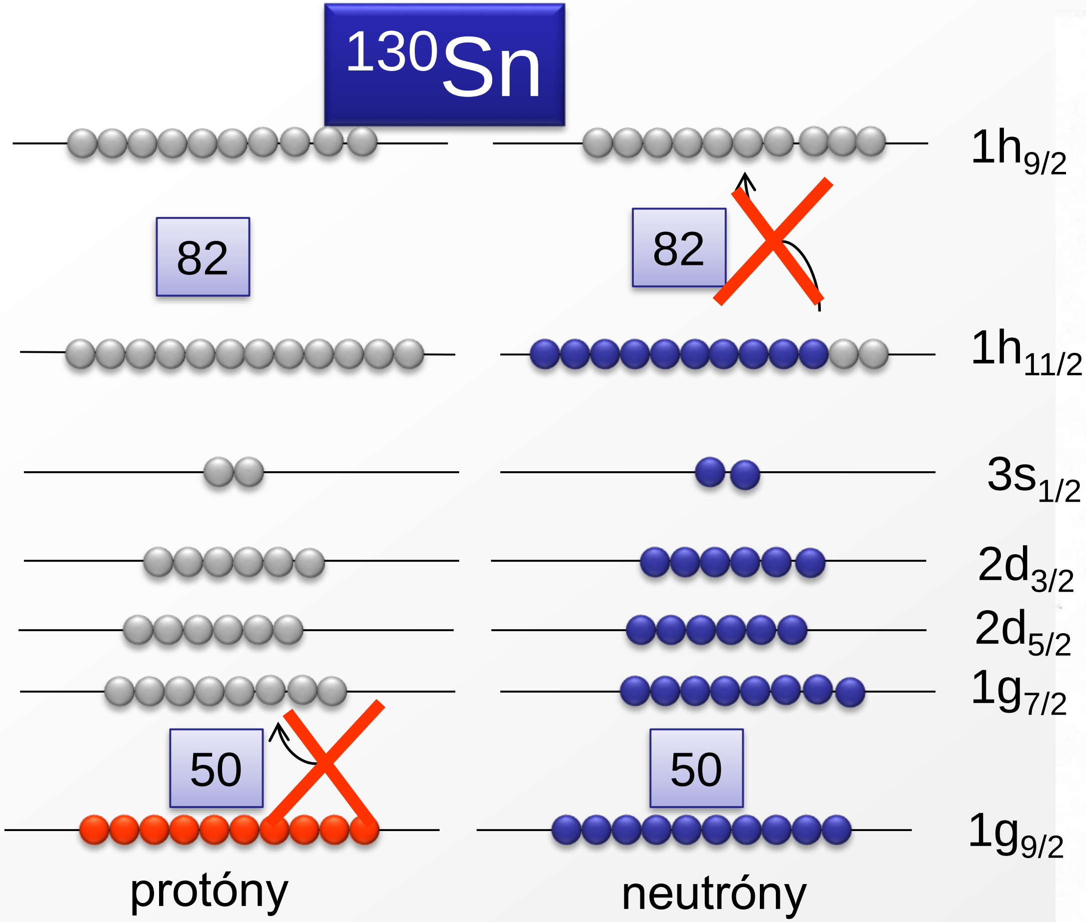


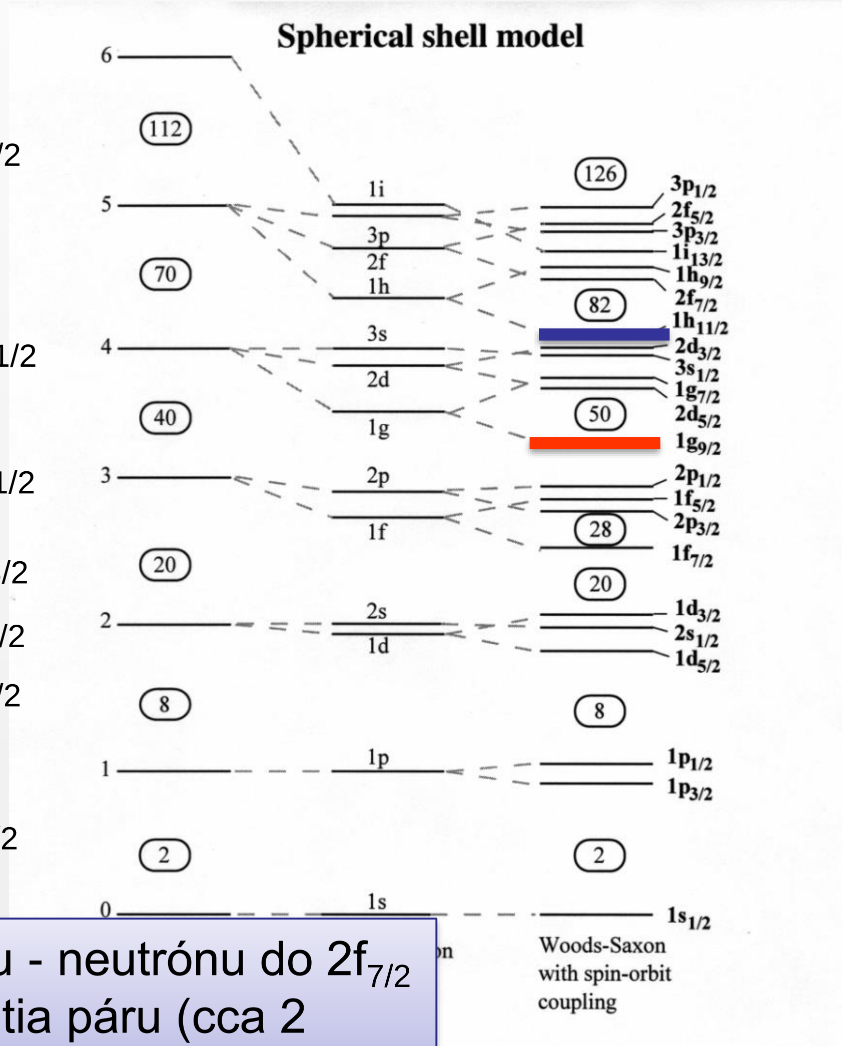
Základný stav 0+

Protóny – zaplnený orbital $1g_{9/2}$

Neutróny – dva chýbajú do zaplnenia orbitalu $1h_{11/2}$

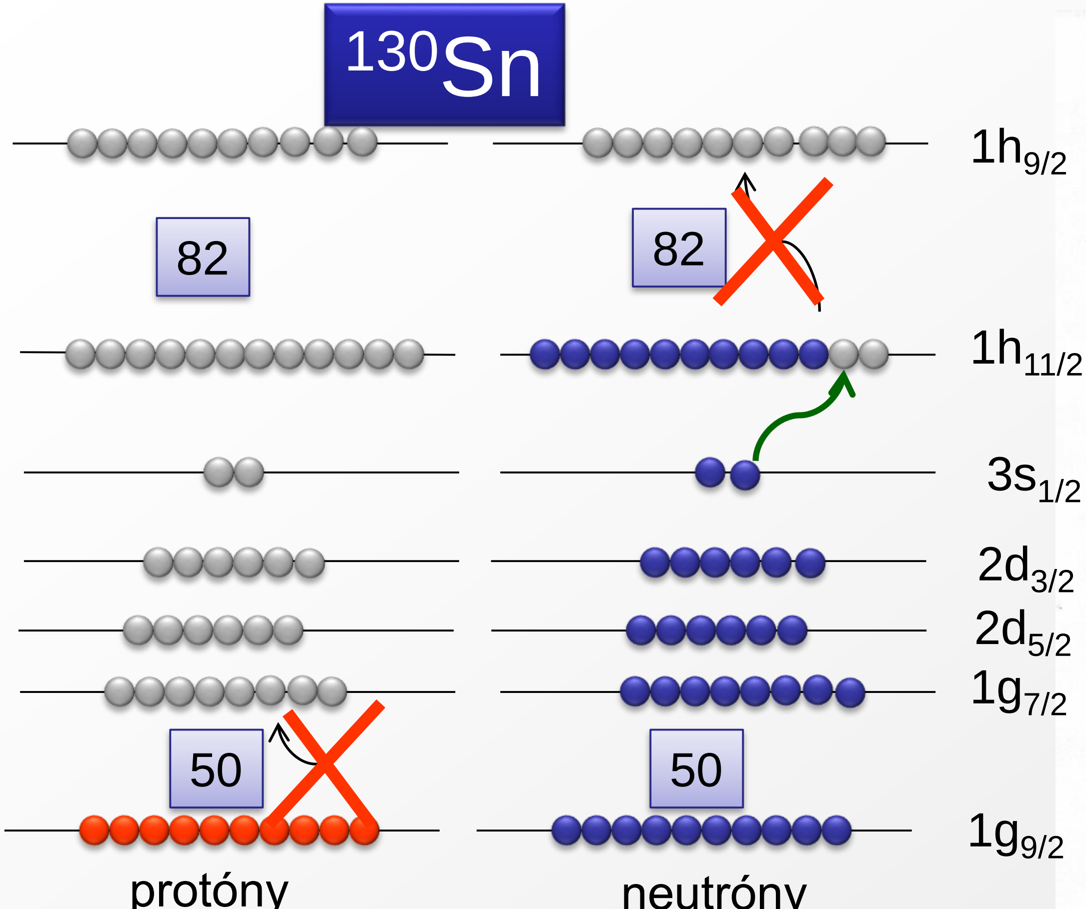
Even-even nuclei - excitation

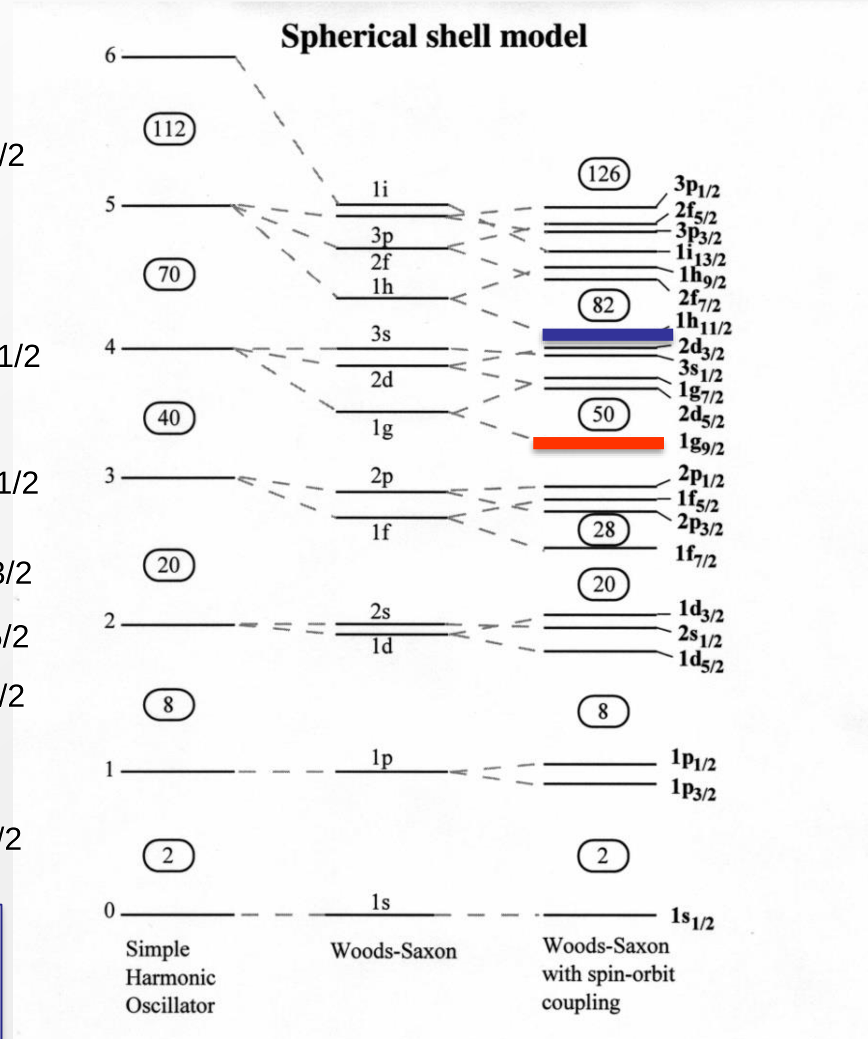




Prenos valenčného nukleónu do ďalšieho orbitalu - neutrónu do $2f_{7/2}$ alebo protónu do $1d_{5/2}$ - vyžaduje okrem roztrhnutia páru (cca 2 MeV) veľkú energiu kvôli medzere medzi vrstvami (pár MeV)

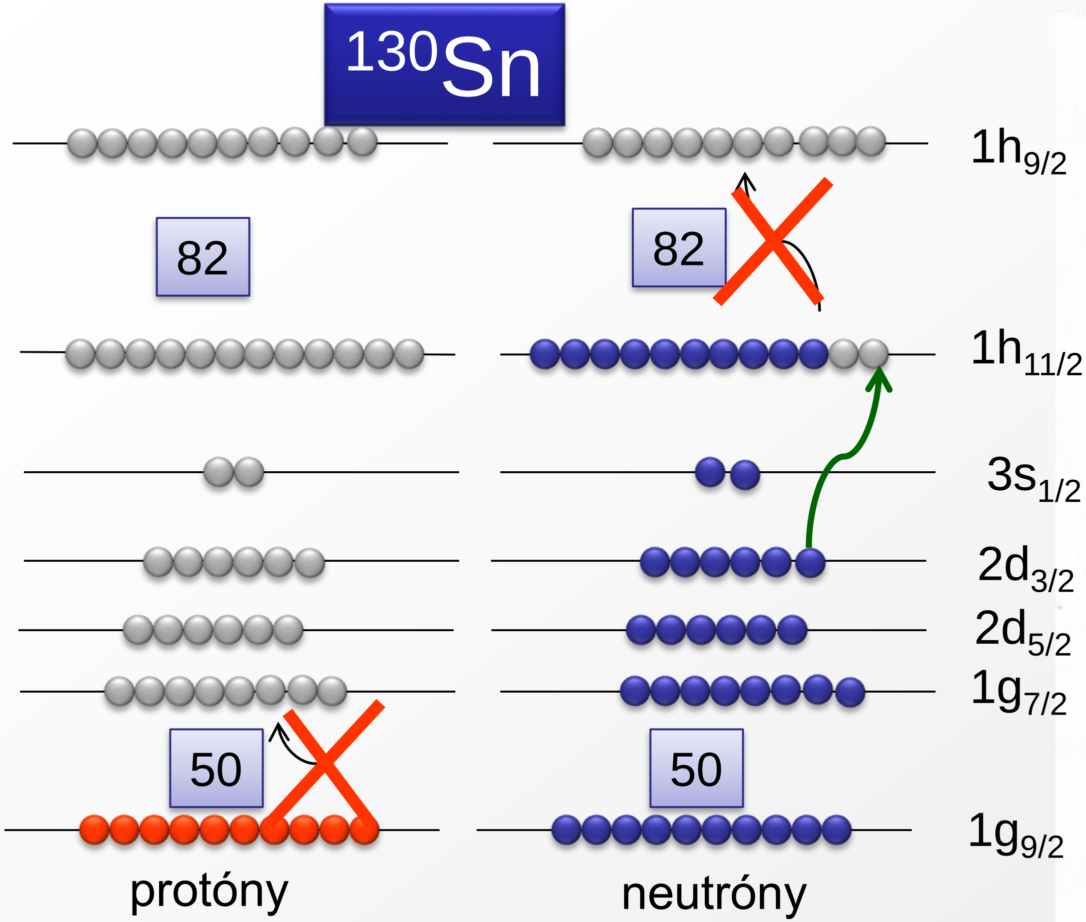
Even-even nuclei - excitation

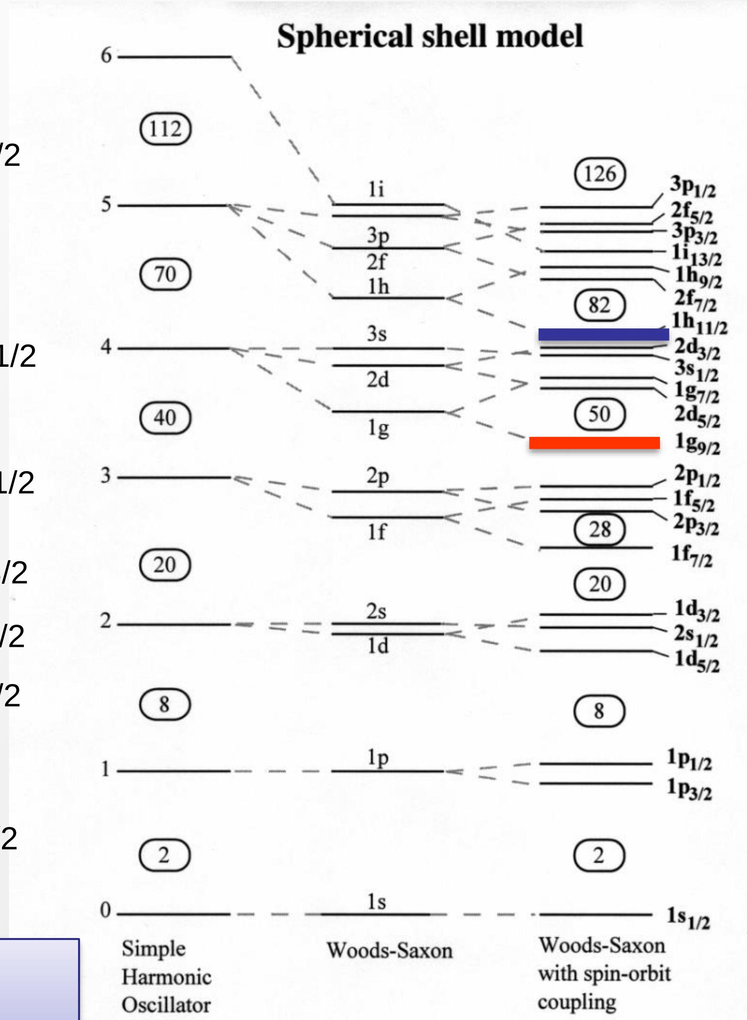




Pozn. $1h_{11/2}$ má zápornú paritu $3s_{1/2}$ kladnú
Vzbudenie z $3s_{1/2}$ potom vedie na stav 6^- (komb.11/2+1/2) a stav 5^- (komb. 11/2-1/2)

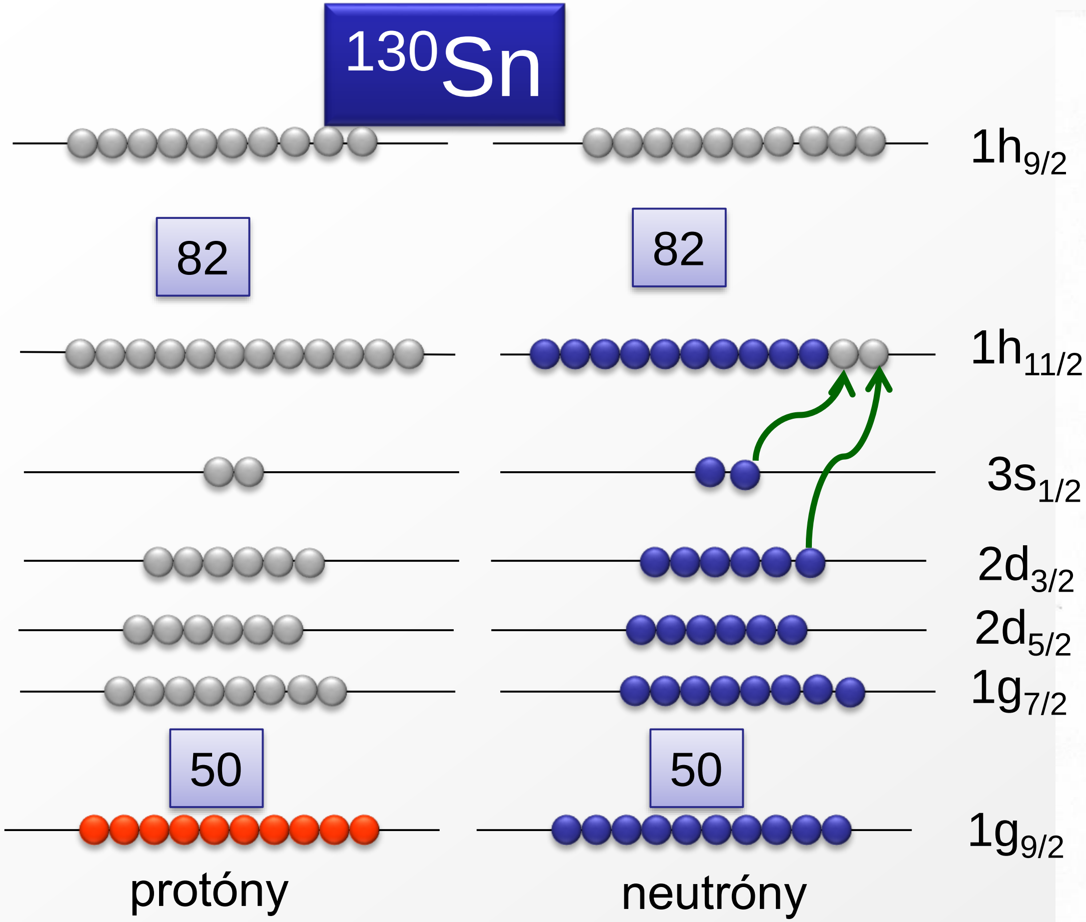
Even-even nuclei - excitation

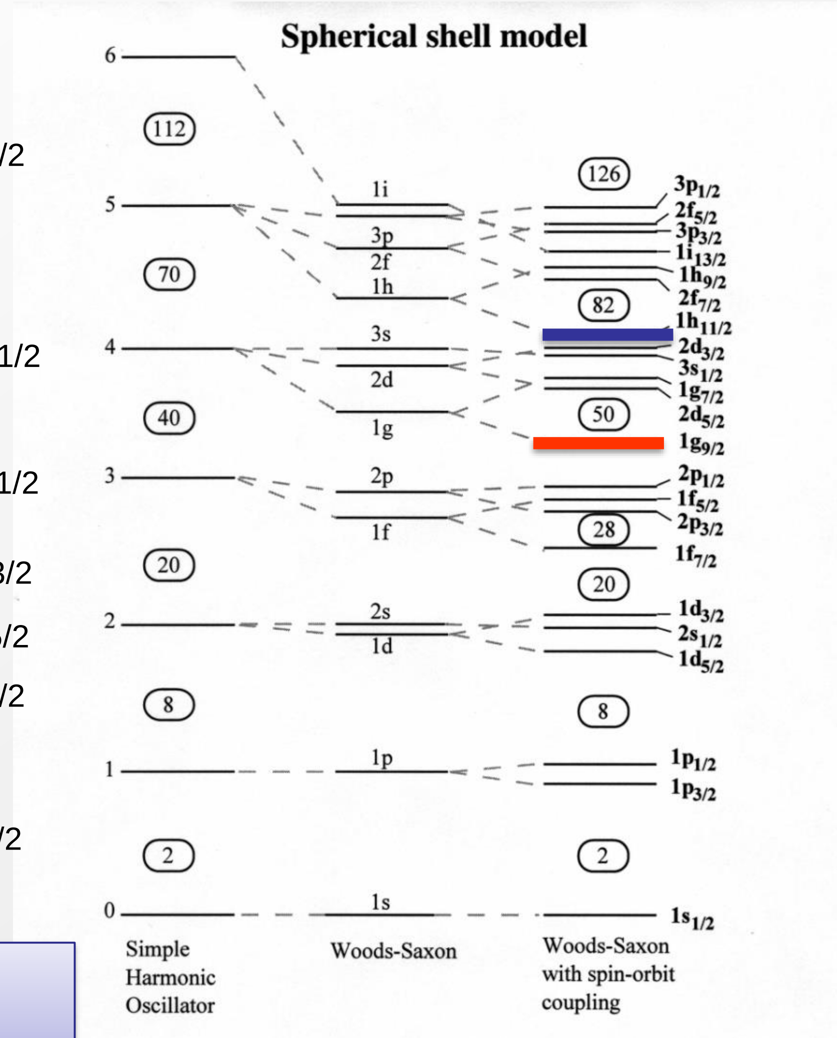




Vzbudenie z 2d$_{3/2}$ potom vedie na stavy od 4$^-$ (komb.11/2-3/2) po stav 7$^-$ (komb. 11/2+3/2)

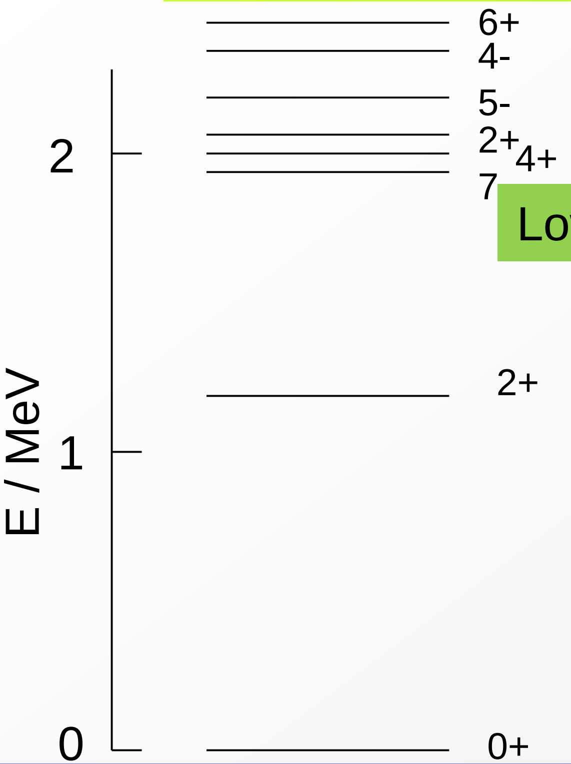
Even-even nuclei - excitation



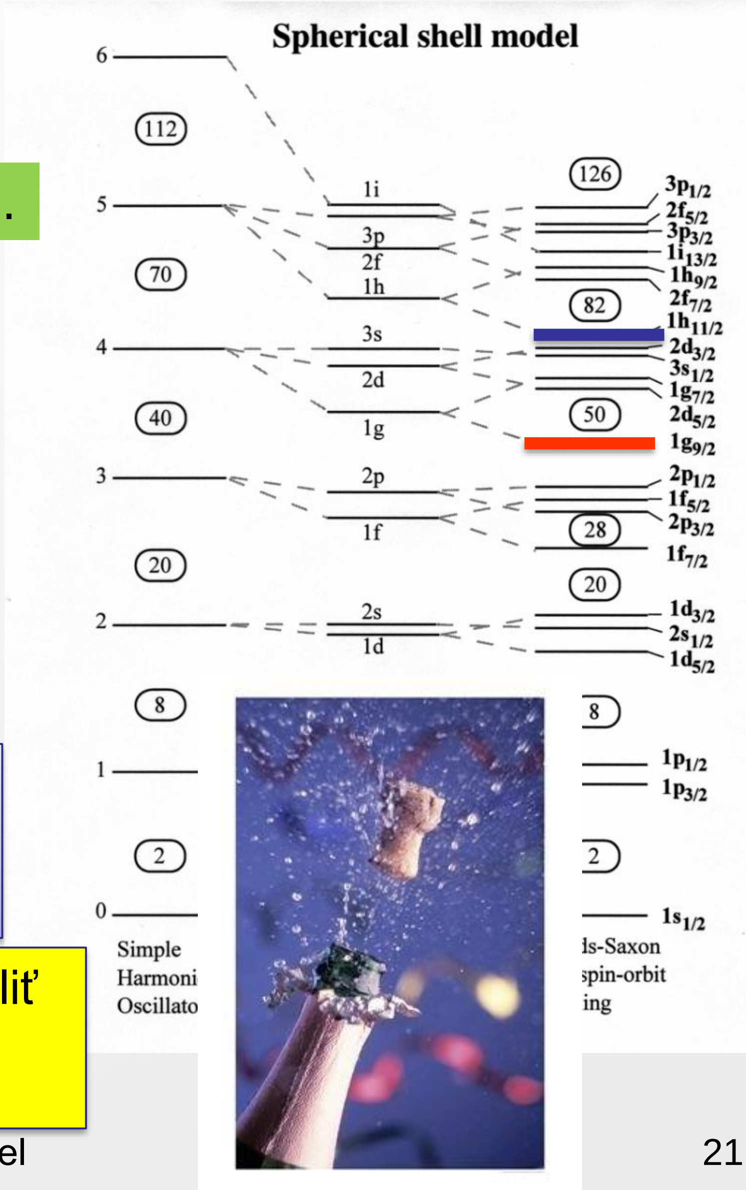


Možností je viacero .. Ako napr. 2+ stav pri prenose neutrónu z $3s_{1/2}$ a $2d_{3/2}$

Even-even nuclei



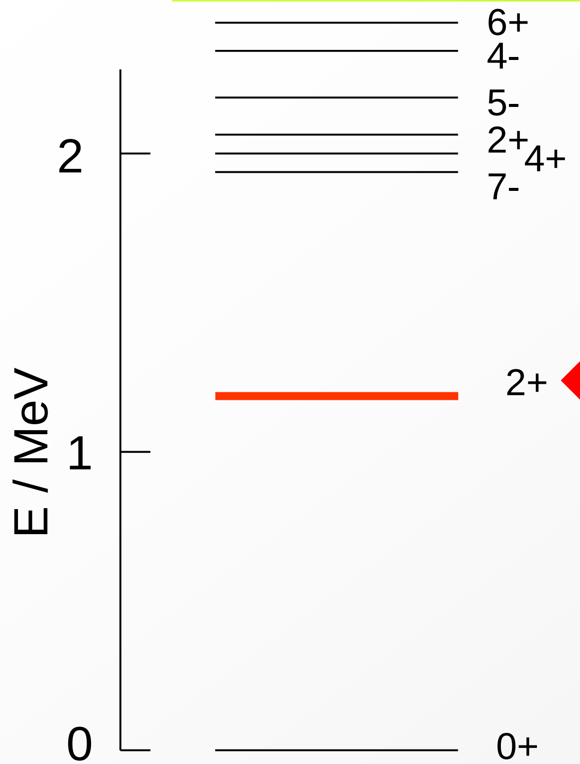
Low-lying states in ^{130}Sn.



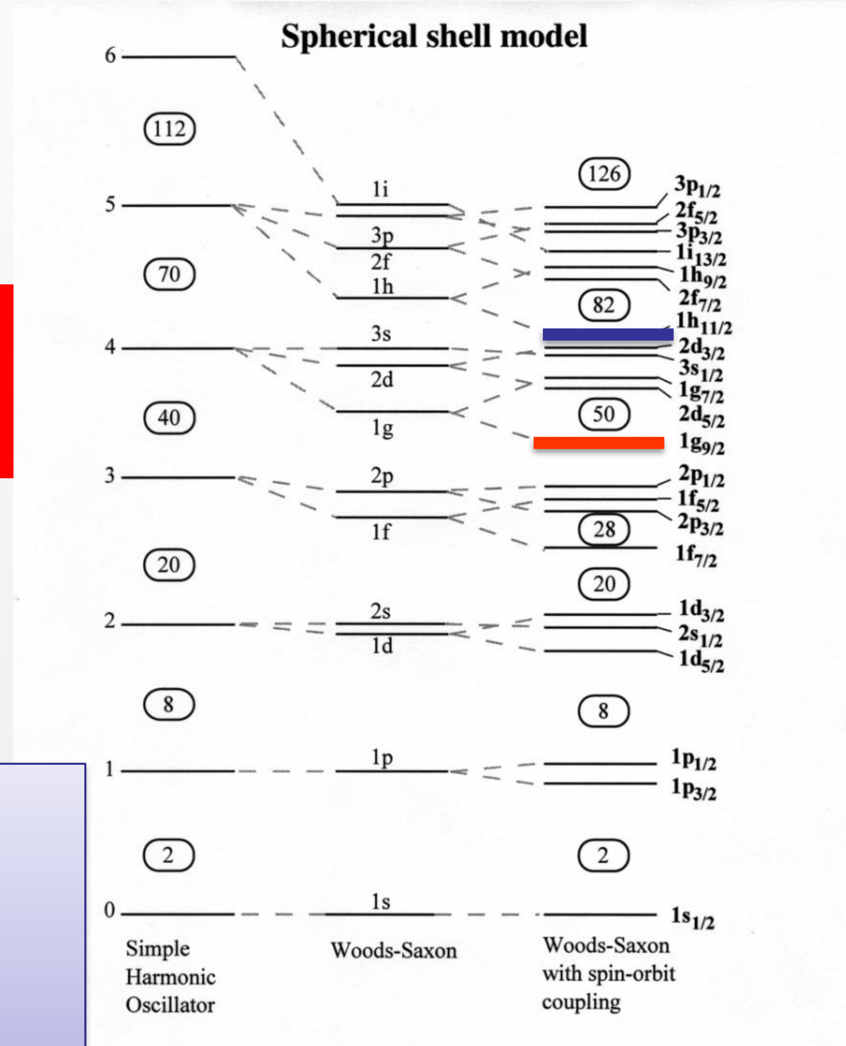
To separate the pair is needed an energy of 1.5-2 MeV. Indeed all deduced levels from previous slides are at 2 MeV and more

Vrstvový model dokáže aspoň kvalitatívne vysvetliť viaceré z excitácii v párno-párnych jadrách.
(pozor na mixing stavov)

Párno-párne jadrá



How we could create this level?



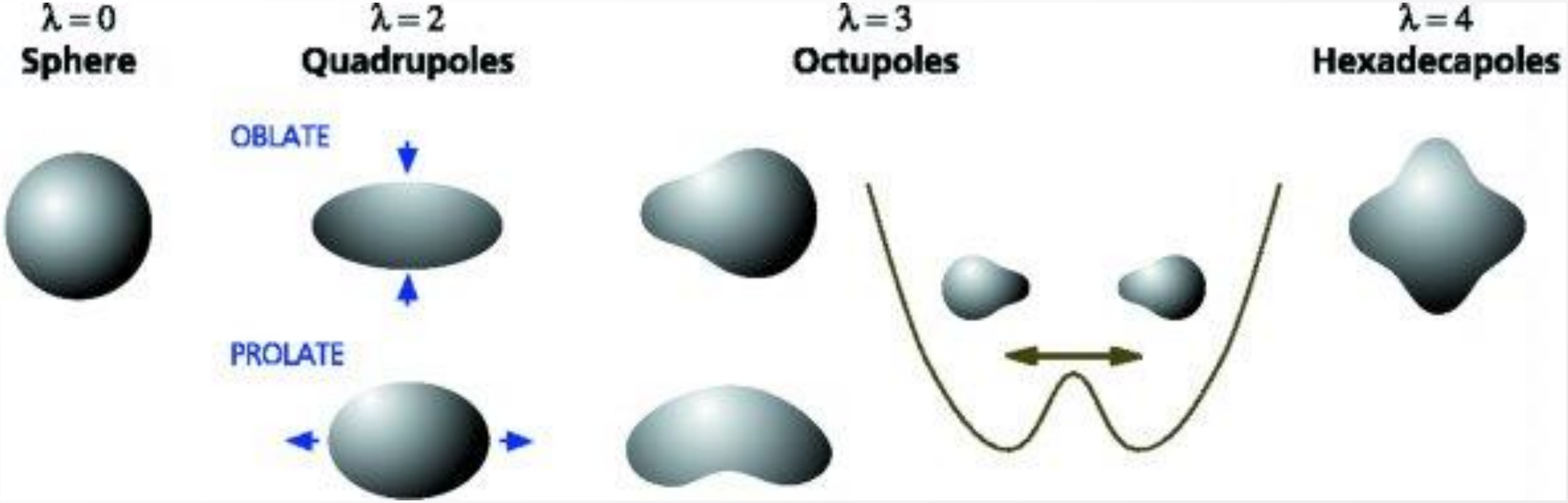
Problem of 2+ state around 1 MeV

It’t not only the case of ^{130}Sn – similar level is almost in any even-even nucleus.

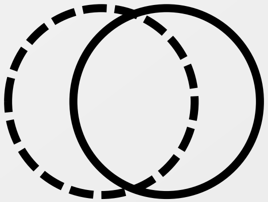
It’s origin is in collective excitations.

NUCLEAR DEFORMATION AND COLLECTIVE EXCITATIONS

Deformácie jadra

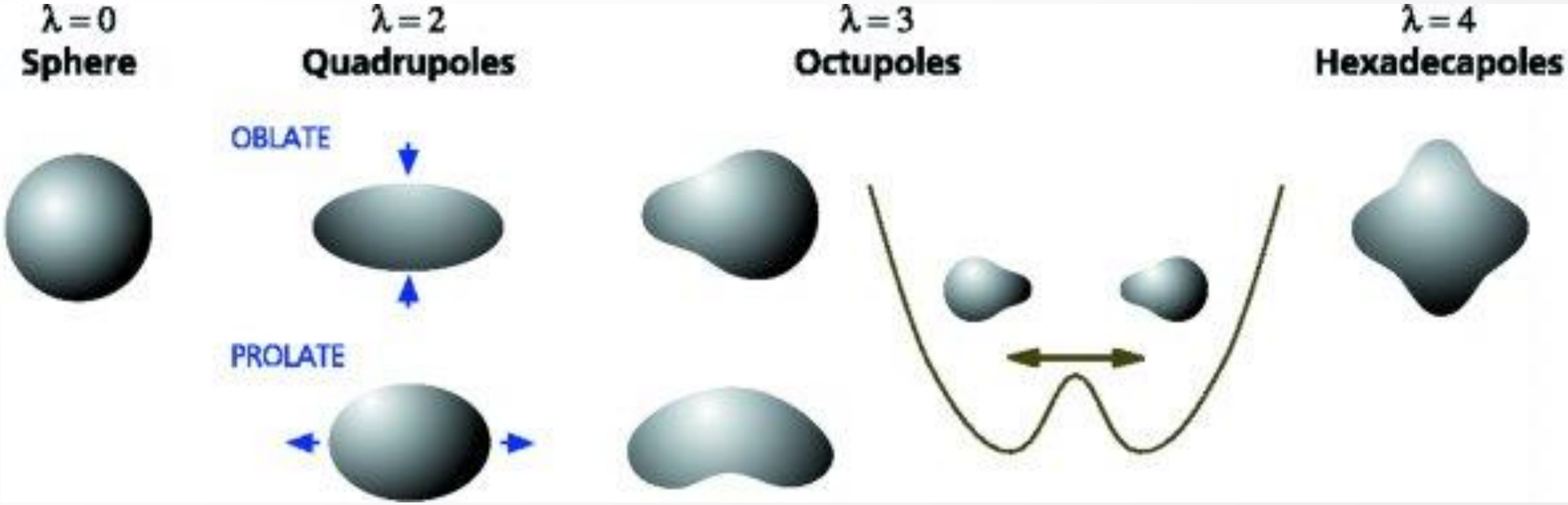


Pozn.: dipólová vibrácia suvisí iba zo zmenou polohy jadra a preto nie je dôsledkom vplyvu vnútorných jadrových síl.

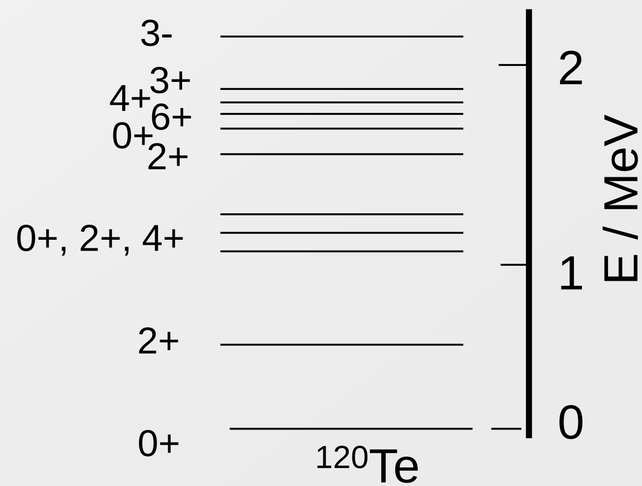


λ = 1 (Dipole)

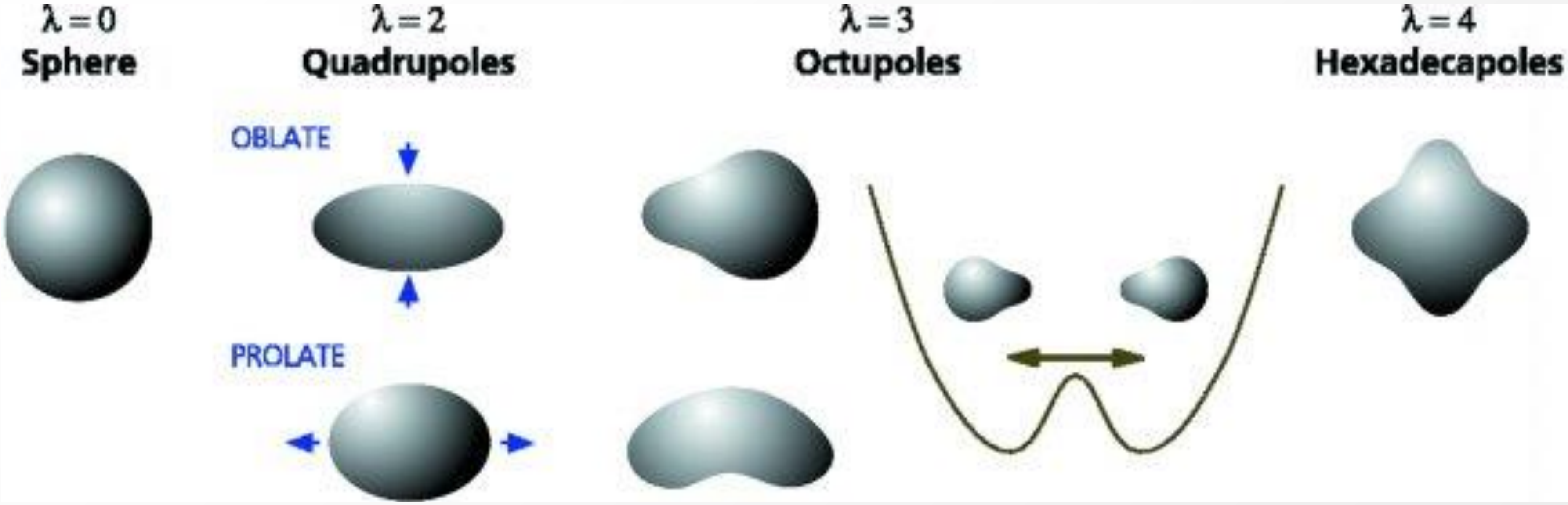
Kvadrupólová deformácie jadra



Pozn. Pre λ = 2 (kvadrupólová deformácia) dostávame trojicu 0+,2+ a 4+ (tzv. „two-phonon triplet“) v cca dvojnásobnej energii prvý 2+ stav a päť stavov 0+,2+,3+,4+,6+ v cca trojnásobnej energii (tzv „three-phonon quintuplet“)



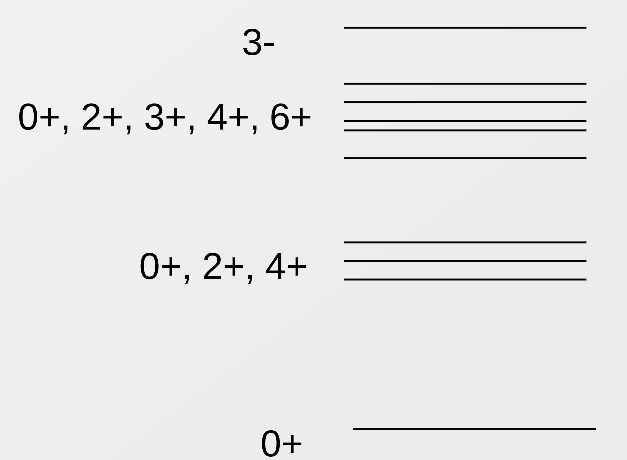
Oktupólová deformácie jadra



λ = 3 (octupólová deformácia) nesie uhlový moment hybnosti ℓ = 3 a teda má negatívnu paritu.

Ako prvý vzbudený stav je hladina 3- vo vibračných jadrách spravidla nad dvoj-fonónovým tripletom.

Pri vyšších energiách (niekoľko MeV) prichádza spravidla k deleniu párov a stavy sa zhusťujú kvoli veľkému množstvu možných kombinácií

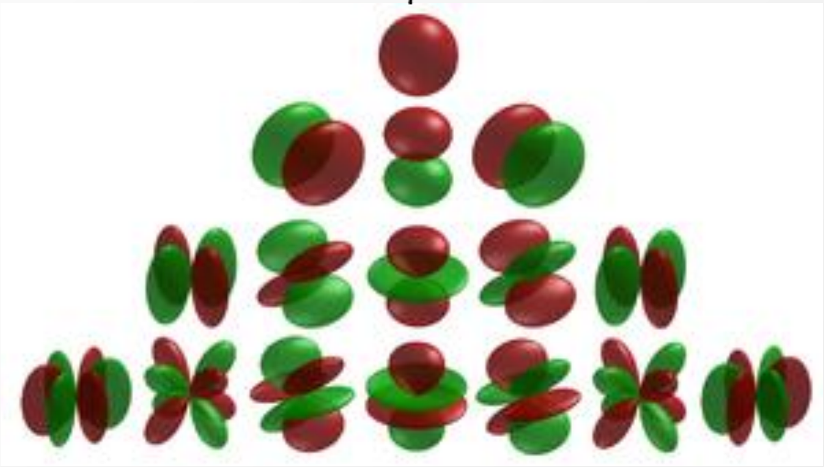


Tvary jadier

Okamžitý tvar jadra sa dá vyjadriť funkciou

$$R(t) = R_{av} + \sum_{\lambda \geq +} \sum_{\mu=-\lambda}^{\check{}+\lambda} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}(\theta, \phi)$$

Kde R_{av} je priemerný polomer jadra, $\alpha_{\lambda\mu}(t)$ je amplitúda sférickej harmonickej funkcie $Y_{\lambda\mu}(\theta, \phi)$ (riešenia Laplaceovej rovnice).

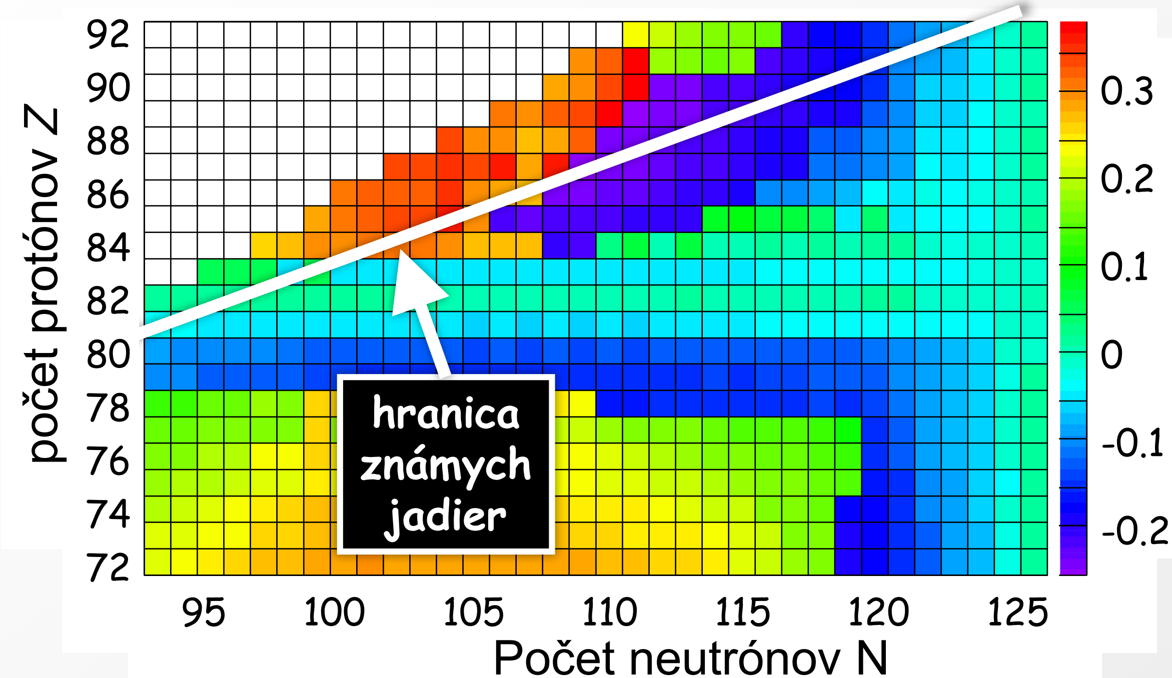


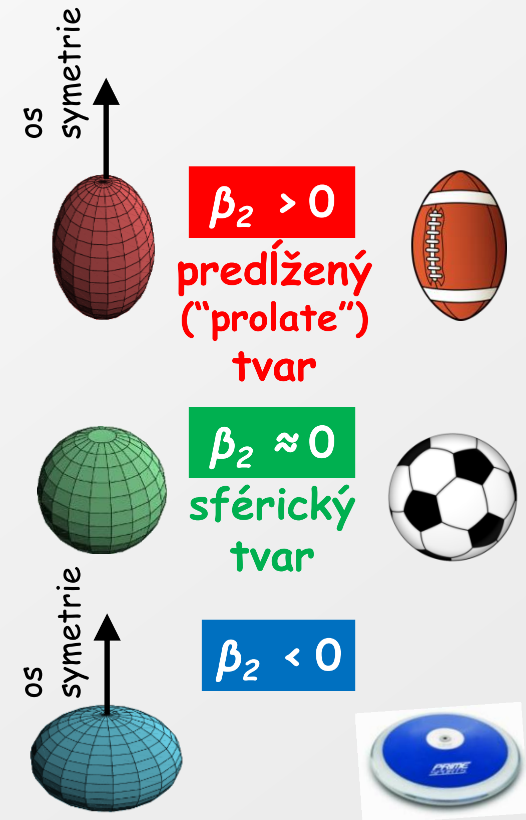
Grafická reprezentáca prvých sférických harmoníckých funkcií (červená zodpovedá kladným hodnotám a zelená záporným)

Deformácie jadier

Kvadrupólová deformácia základného stavu jadra β_2

Výpočty sú založené na FRDM a folded-Yukawa single-particle microscopic modeli [P. Möller *et al.*, *At. Data Nucl. Data Tables* **59**, 185 (1995)].

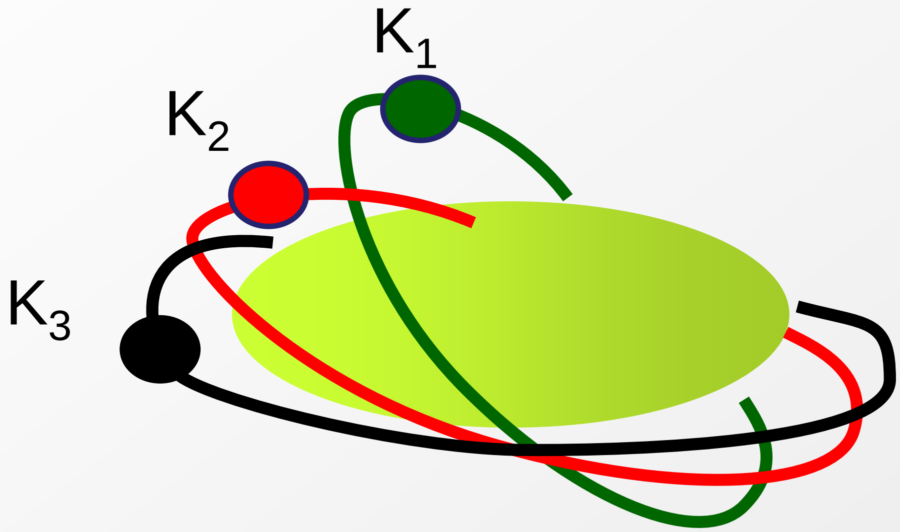




DEFFORMED SHELL (NILSSON) MODEL

Deformed shell– Nilsson - model

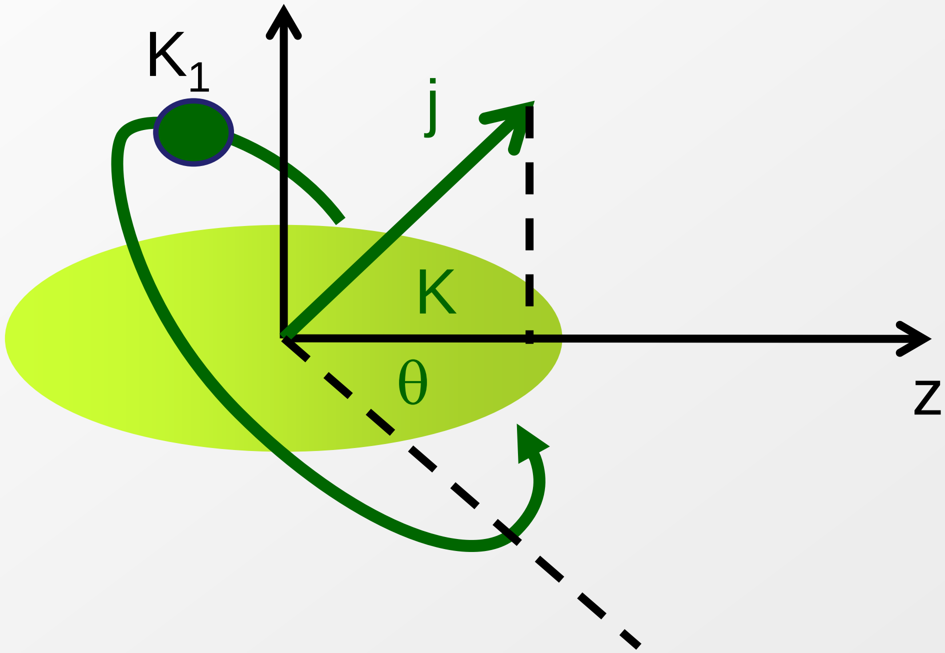
Nilsson model describes the single-particle situation in the deformed potential. The particle could have different angle of its orbital with the symmetry axis.



Deformovaný – Nilssonov model

The nucleon move within an orbital in the plane with the defined angle to the symmetry axis θ.

We could make a simple guess and write $\sin\theta = \frac{K}{j}$. The possible K numbers (projections) could have values 1/2, 3/2, 5/2...

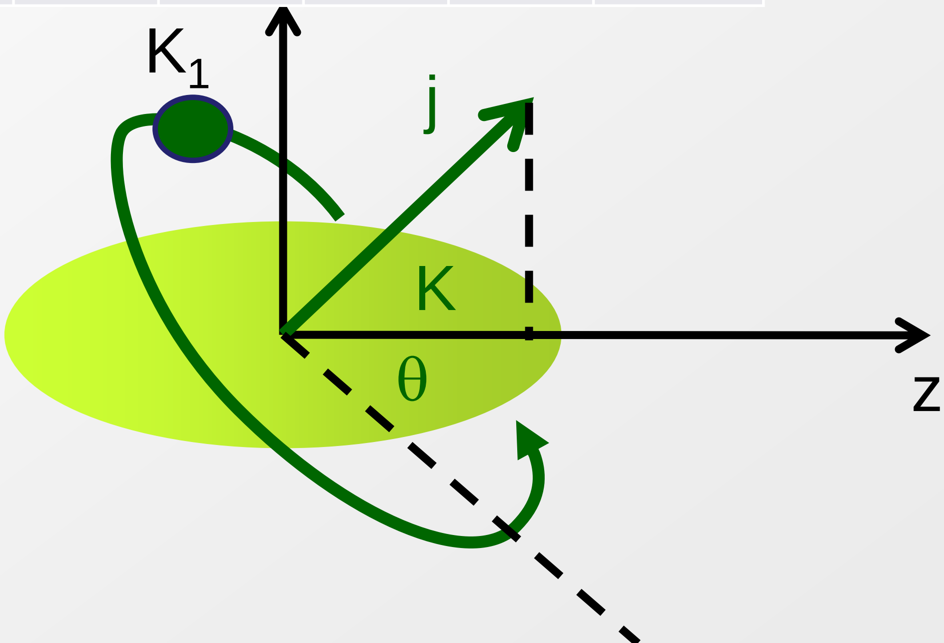


Závislosť od uhla roviny orbitalu

Zaujímavé je, že uhol naklonenia θ by sa mal meniť pomalšie pre menšie hodnoty priemetu K a rýchlejšie pre väčšie.

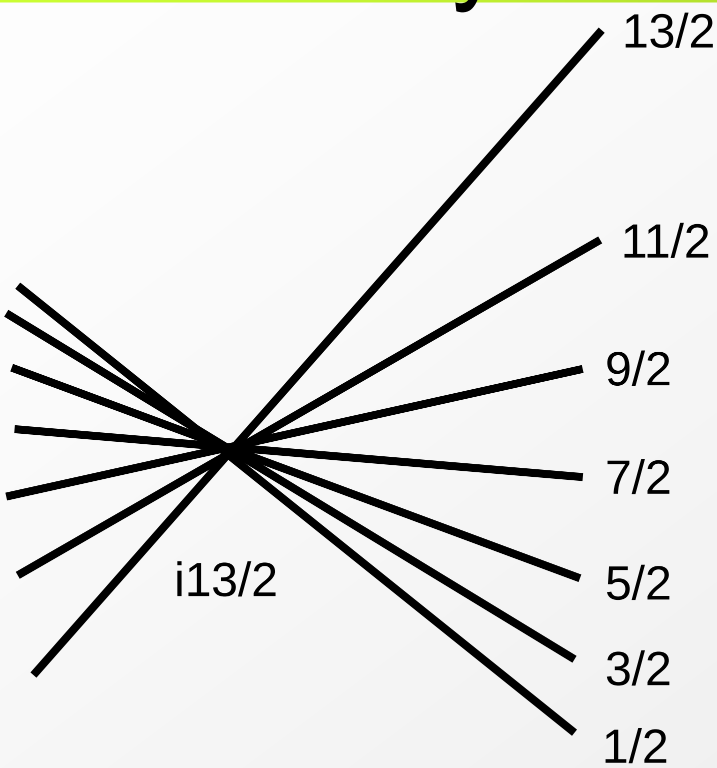
Konkrétne možno napísať $\theta = \sin^{-1}\frac{K}{j}$. Príklad pre j=13/2.

K	1/2	3/2	5/2	7/2	9/2	11/2	13/2
θ(deg)	4.4	13.3	22.6	32.6	43.8	57.8	90
Δθ(deg)	8.9	9.3	10	11.2	14.0	32.2	



Zmena energie pre malé hodnoty K, resp. Ω, je menšia ako pre veľké hodnoty.

Zmena energie pre malé hodnoty deformácie



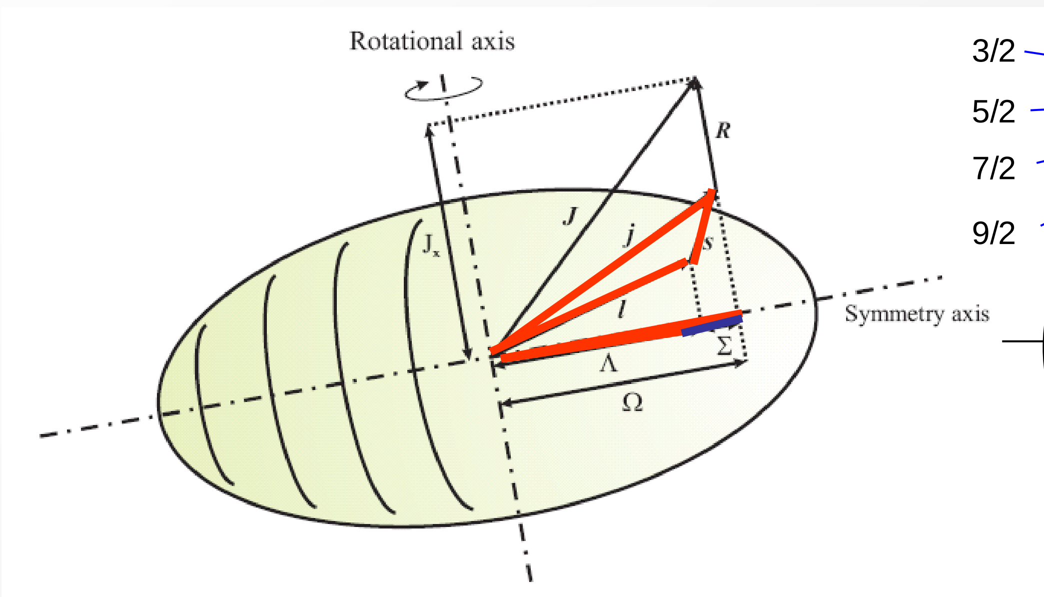
Takto si môžeme jednoducho odhadnúť správanie sa hladín pre malé deformácie.

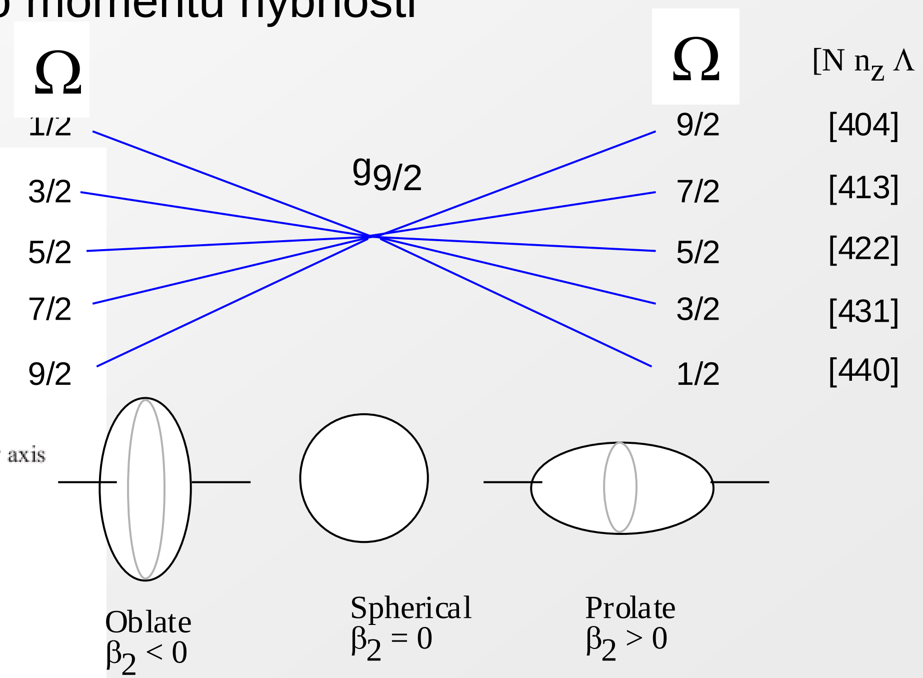
Ďalšia zmena smeru nastáva až keď sa blíži hladina k inej s rovnakým K a paritou.

Prípad deformovaného jadra

Zavádzajú sa nové tzv. asymptotické kvantové čísla.

j - moment hybnosti častice (orbitálny moment hybnosti plus spin)
Ω - projekcia momentu hybnosti do osi symetrie (j_z)
Σ, Λ – projekcie orbitalneho a spinoveho momentu hybnosti
n_z – kvantové číslo oscilátora





Označenie hladín

Plné označenie hladín v Nilssonovom modeli nám posktuje informáciu o hlavnom kvantovom čísle oscilátor aj priemete orbitálneho momentu hybnosti

Označenie hladiny $\Omega^{\pi}[N\ n_z\ \Lambda]$

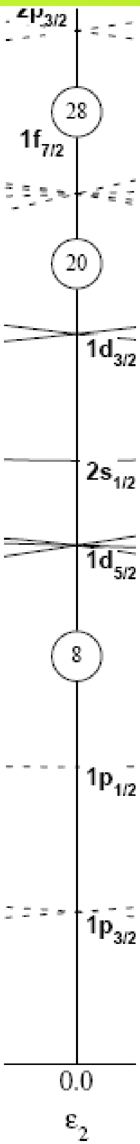
N=6	4s, 3d, 2g, 1i
N=5	3p, 2f, 1h
N=4	3s, 2d, 1g
N=3	2p, 1f
N=2	2s, 1d
N=1	1p
N=0	1s

Hlavné kvantové číslo oscilátora *N* nám taktiež nesie informáciu o parite.

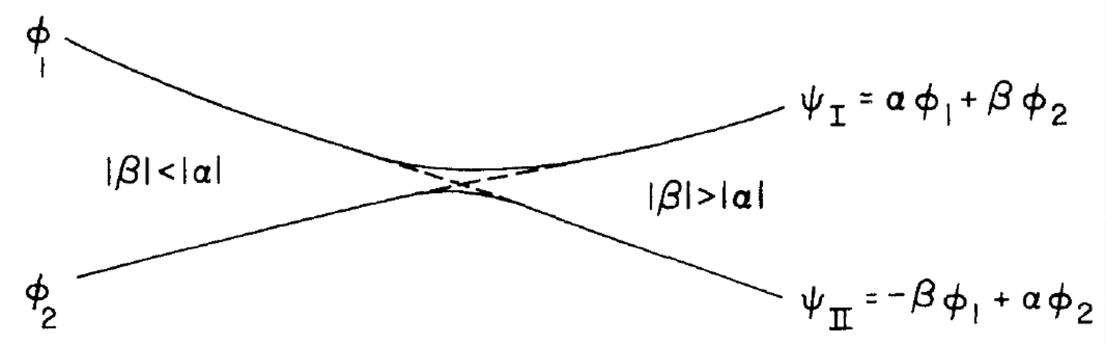
n_z môže maximálnu hodnotu dosiahnuť práve hodnotu *N*

Takže napr. $\Omega^{\pi}[N\ n_z\ \Lambda]$ = 1/2-[550] zodpovedá hladine z orbitalu h11/2. Maximálny priemet do osi symetrie je 5 (l=5 zodpovedá v spektroskopickej notácii písmenu “h”) a hlavné kv. číslo oscilátora 5 nám indikuje 1h.

Nilssonov diagram Z<28

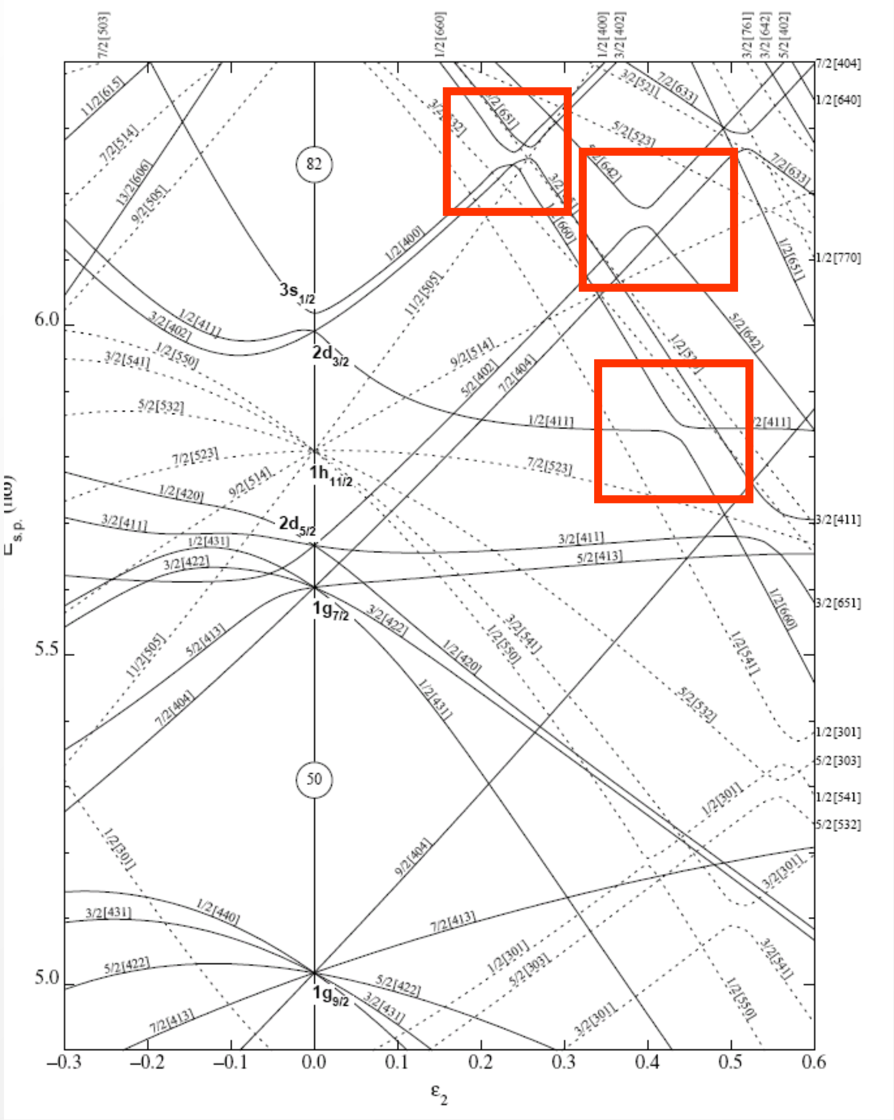


Interakcia orbitalov

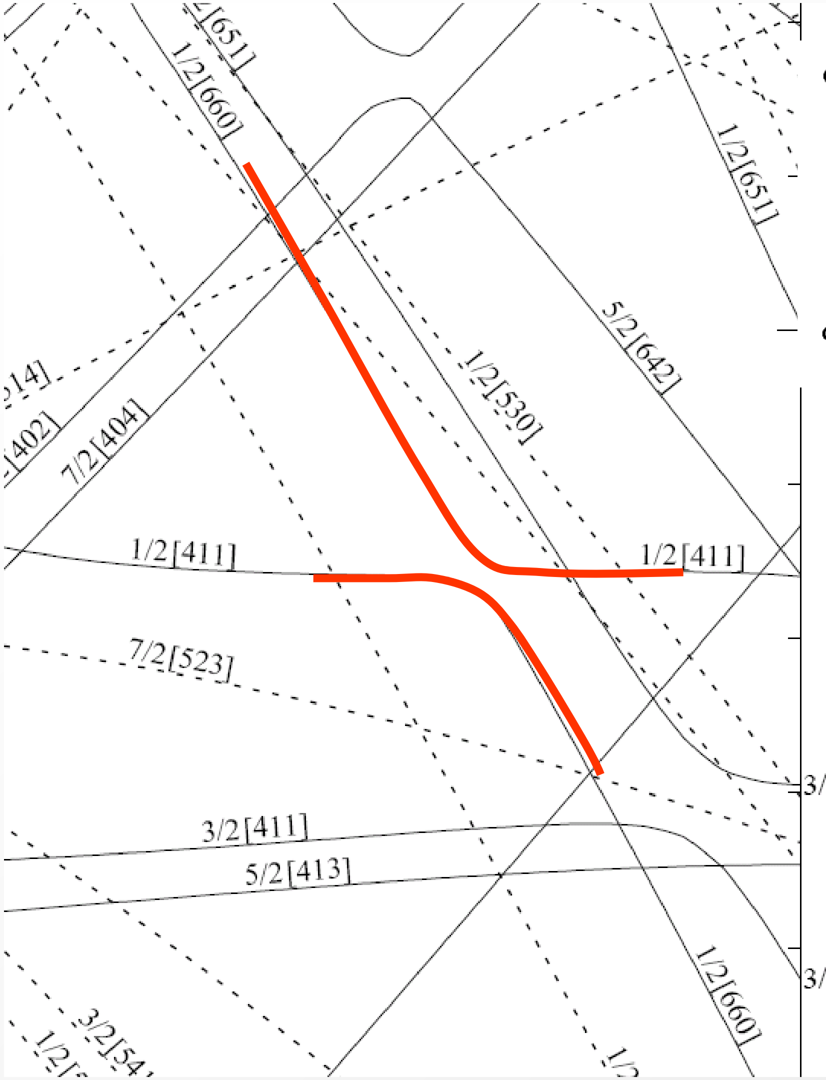


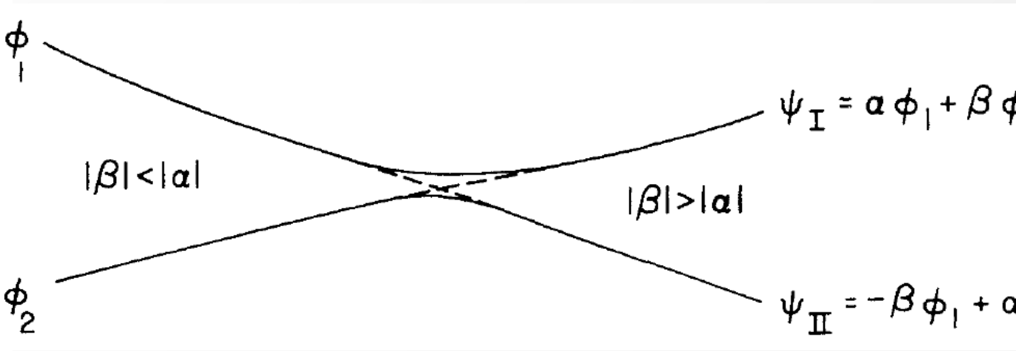
Predpokladajme dva stavy 1 a 2,ktorých energie závisia od parametra x jadrovej štruktúry. Napr. x môže byť parameter deformácie. Predpokladajme existenciu x_{crit} v ktorom by sa mali pretnúť. Energetické hladiny týchto stavov sa začnú miešať a efektívne sa budú odpudzovať a v ďalšom vývoji sa budú miešať. Bod v ktorom sa najviac priblížia bude zodpovedať bodu, v ktorom budú mať rovnakú prímes každého zo stavov.

Nilssonov diagram 50<Z<82



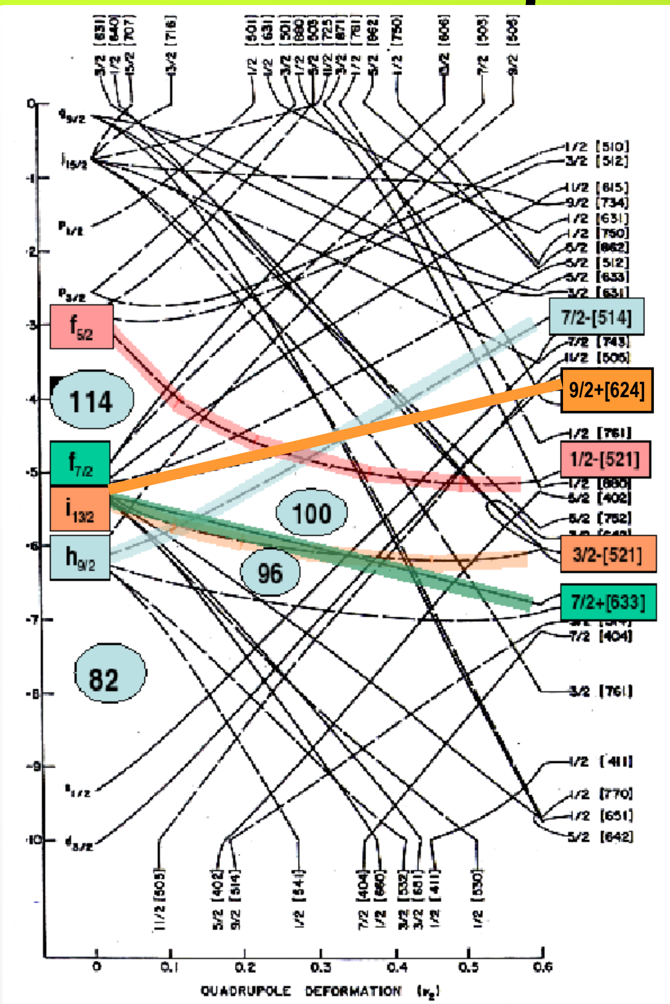
Interakcia orbitalov



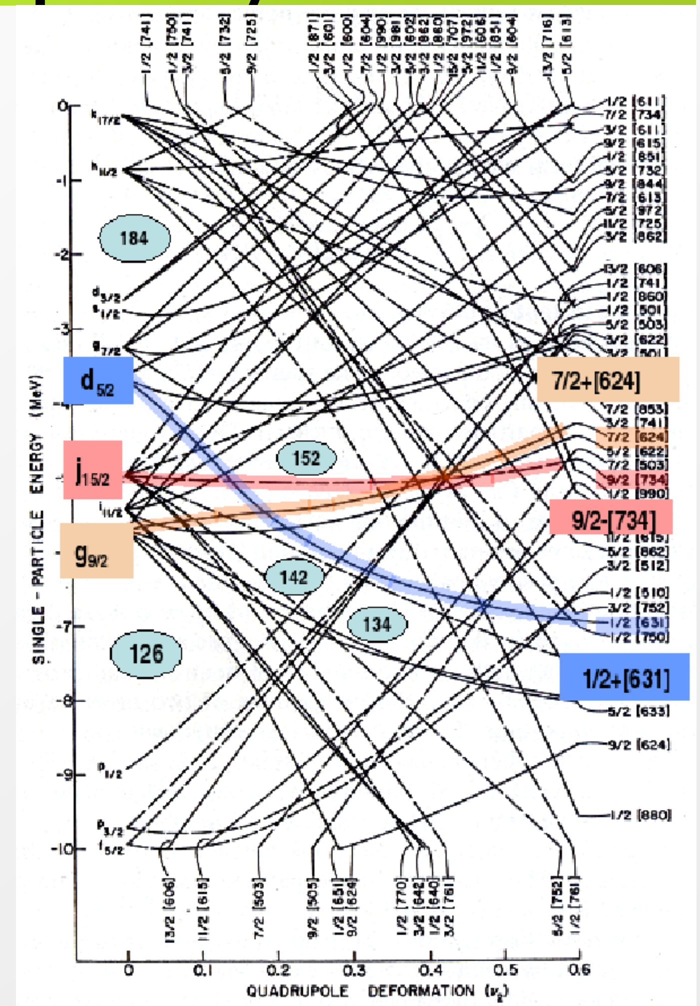


Nilsson diagram
najťažšie prvky

Protons



Neutrons

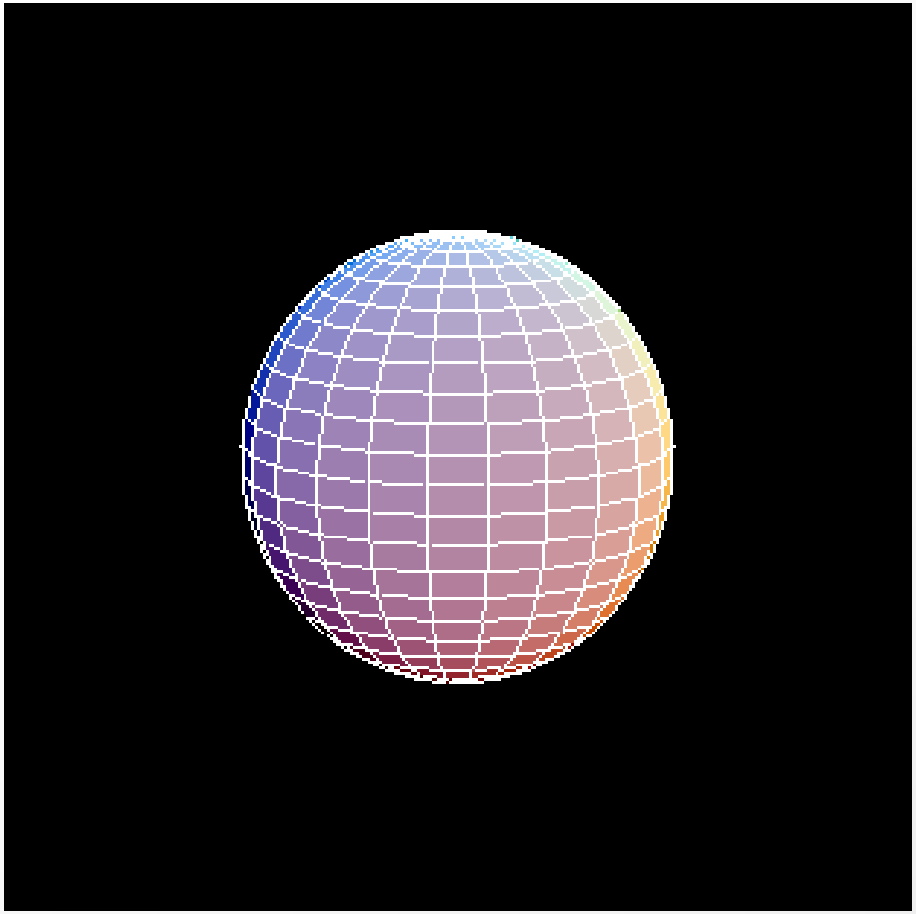


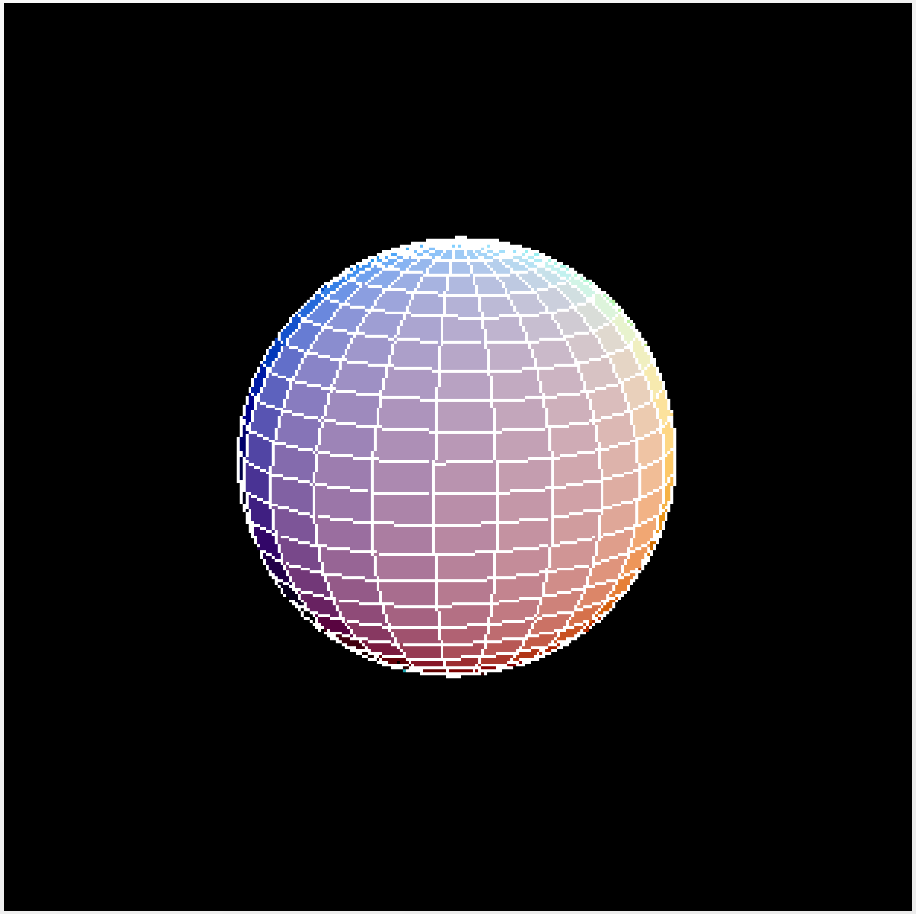
KOLEKTÍVNE EXCITÁCIE JADIER

Kvadrupólové vibrácie jadra

Quadrupólová β vibrácia

Quadrupólová γ vibrácia

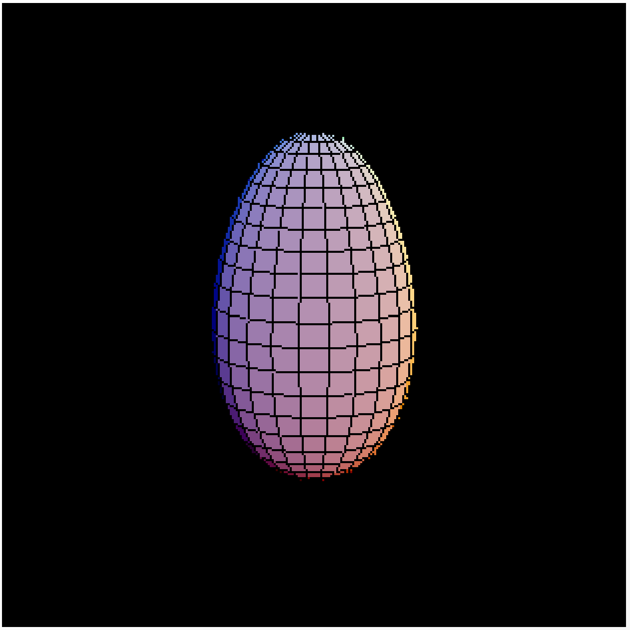




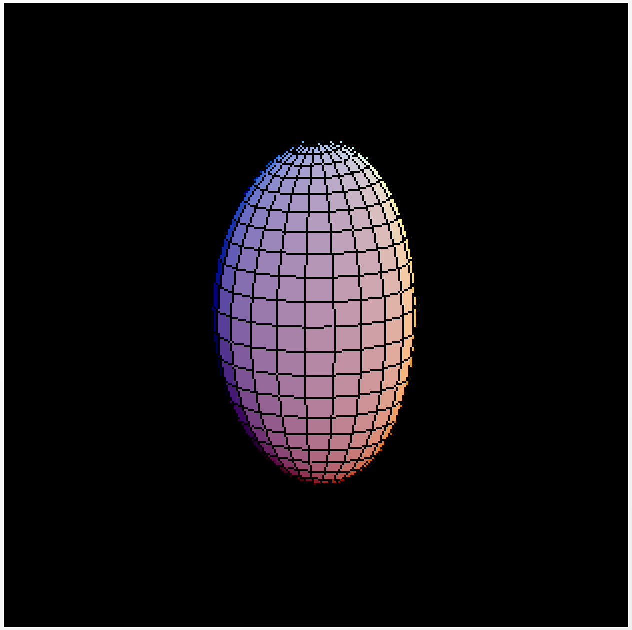
http://radware.phy.ornl.gov/movies/

Oktupólové vibrácie jadra

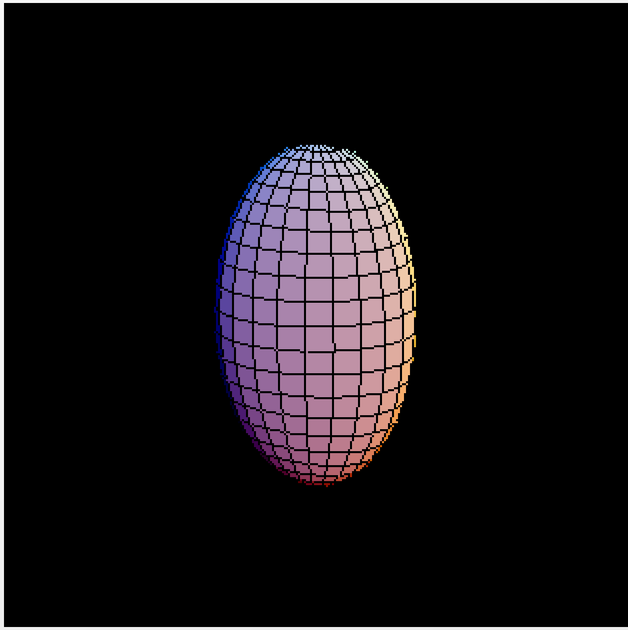
Y30 vibrácia



Y31 vibrácia



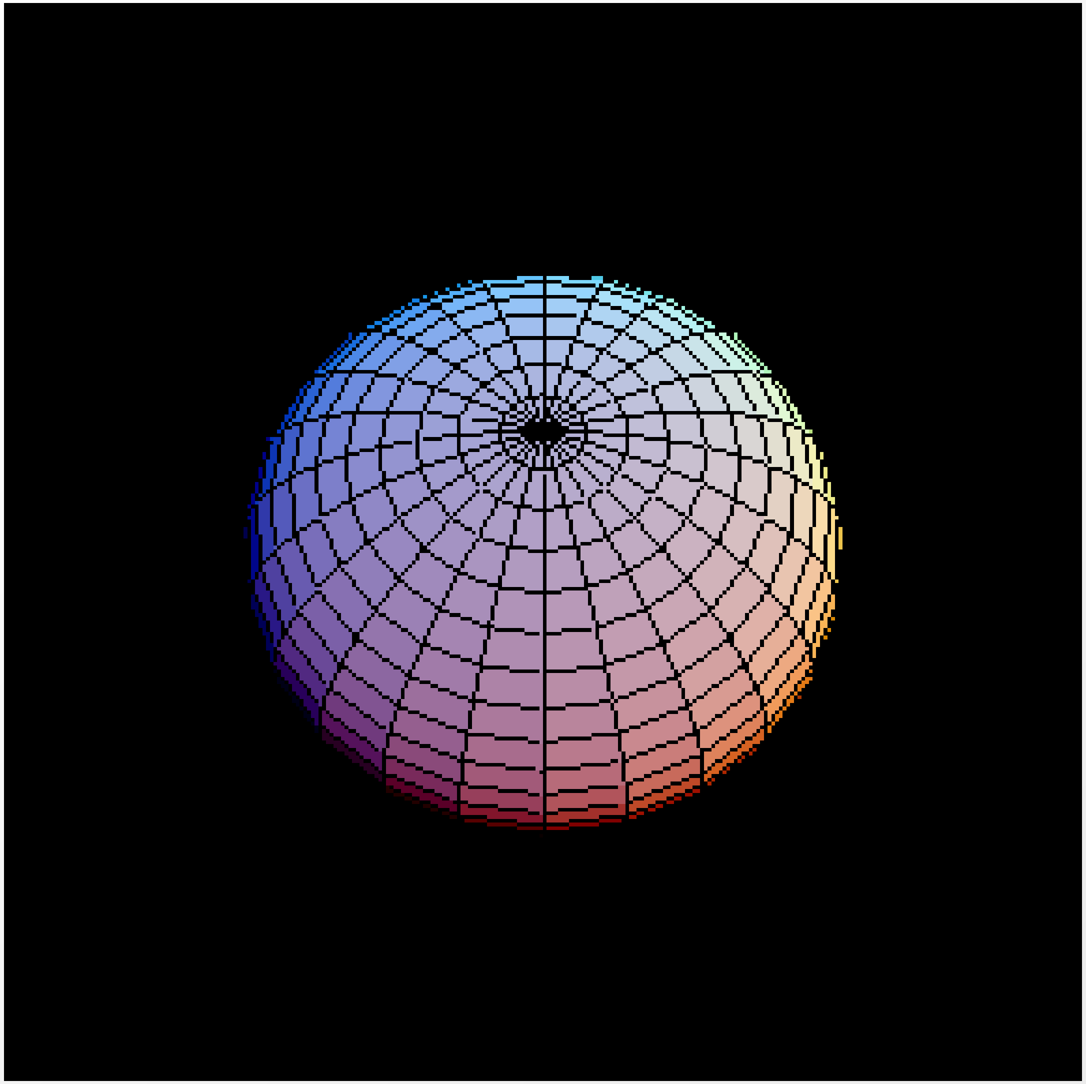
Y32 vibrácia



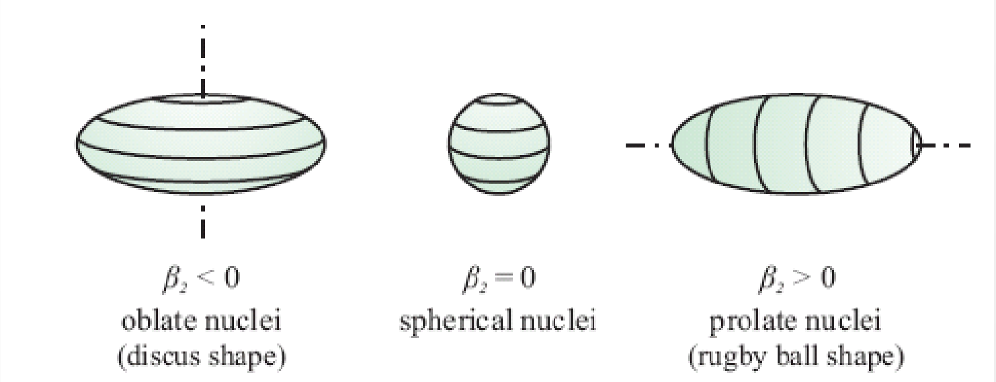
http://radware.phy.ornl.gov/movies/

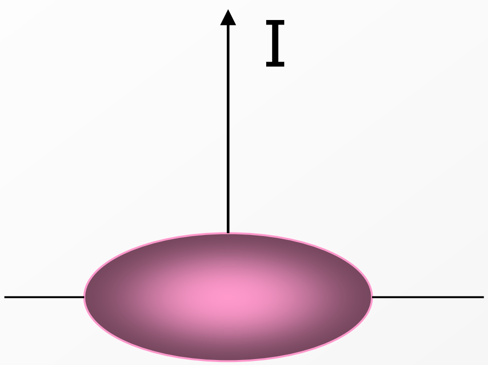
Vibrácia+rotácia

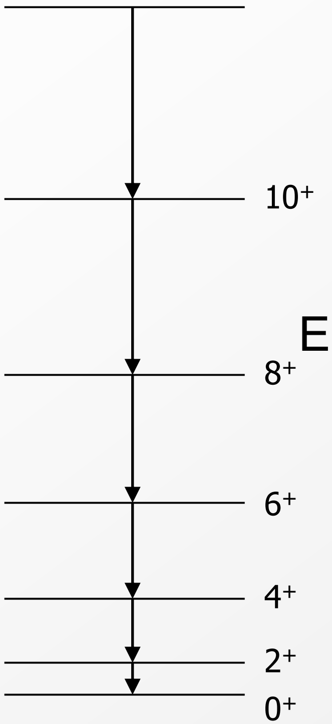
Y32 vibrácia + rotácia



Rotačné stavy







^{156}Dy

I=Jω

$E_I=\eta^2/2J\ I(I+1)$

Energia rotačných stavov

$$E_{rot} = \frac{\hbar^2 J(J+1)}{2\mathcal{J}}.$$

$$E_{rot} = \frac{\hbar^2}{2\mathcal{J}}\left(J(J+1) - K(K+1)\right).$$

Pri vysokých rotáciách aj odstredivé sily

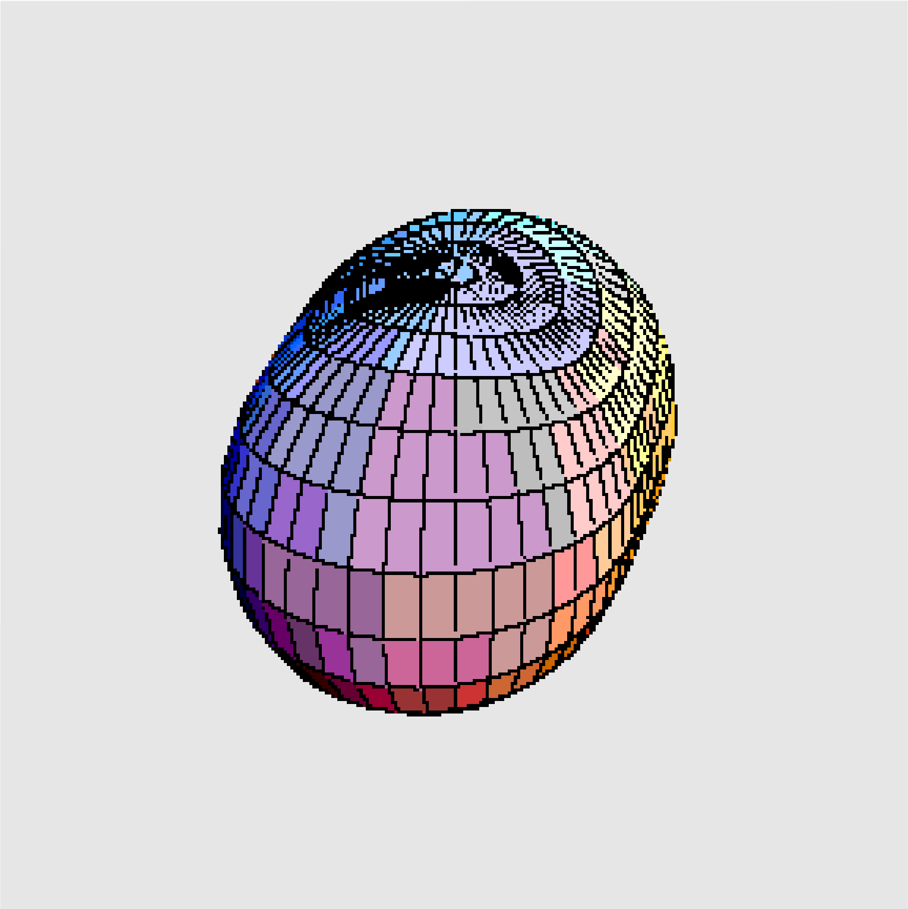
$$E_{rot} = \frac{\hbar^2}{2\mathcal{J}}\left(J(J+1) - \alpha J^2(J+1)^2\right)$$

Tuhé teleso (rigid rotor)

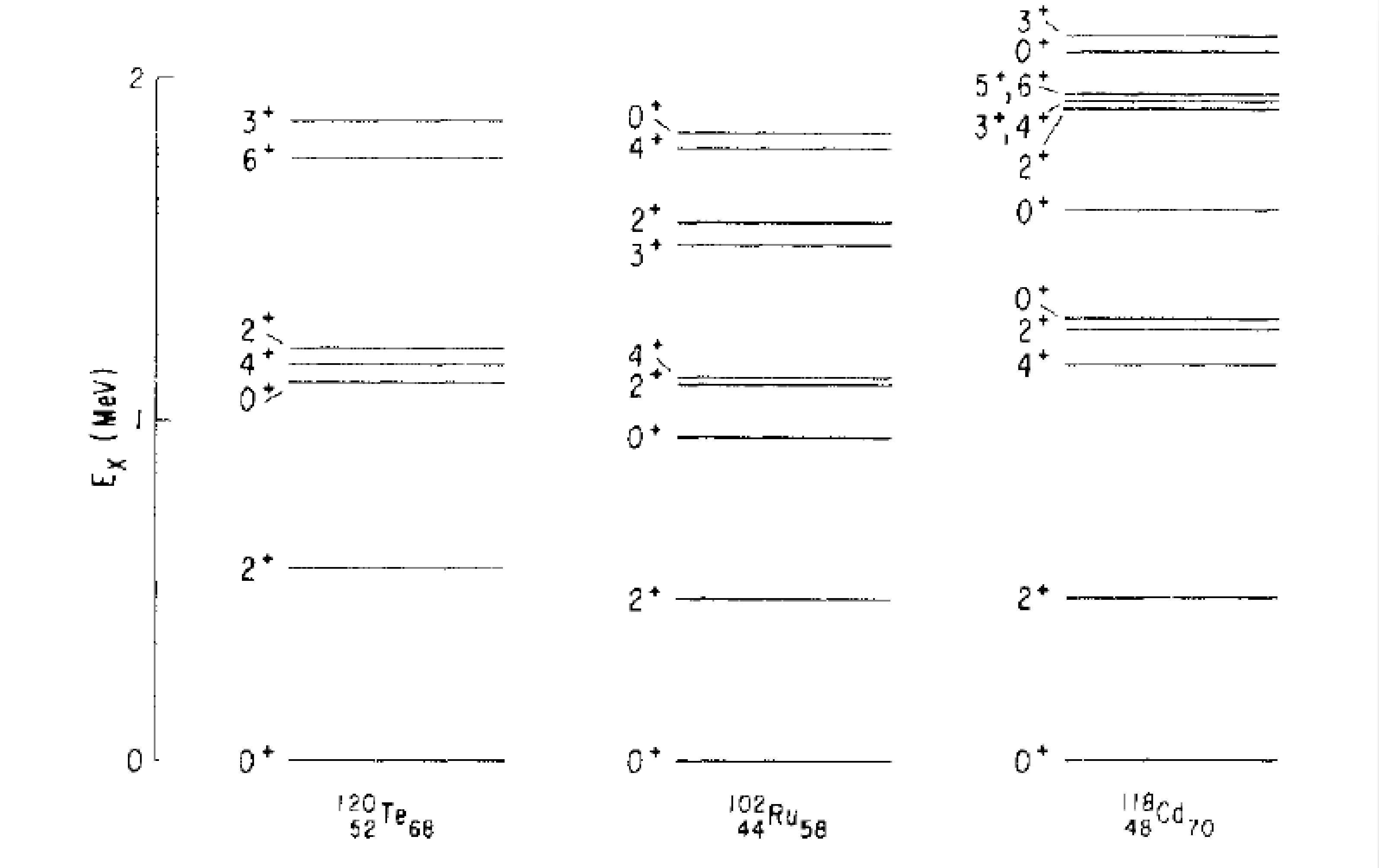
$$\mathcal{J}_{rig} = \frac{2}{5}AMR_0^2(1+0.31\beta),$$

Rotácia jadra

Sférické jadrá nemôžu rotovať. Musí byť definovaná os symetrie.

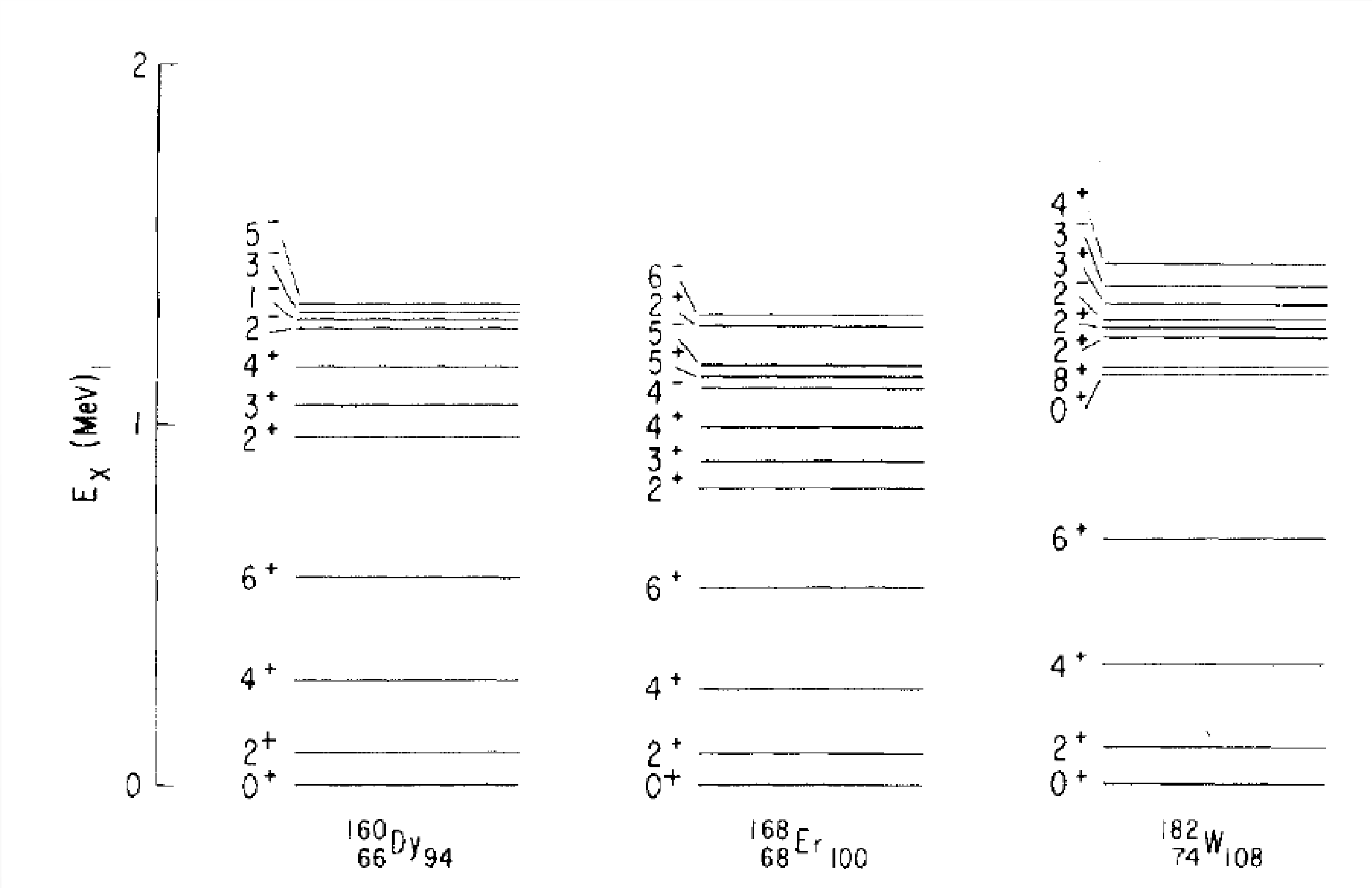


Vibračné stavy



Nízko-ležiace vzbudené hladiny pre vibračné stavy jadier. Relatívne vysoko lokalizovaný prvý 2+ stav nasledovaný dvoj-fonónovým tripletom

Rotačné stavy



Nízko-ležiace vzbudené hladiny pre dobre deformované jadra prvkov z oblasti vzácnych zemín. stavy jadier. Deformácia umožní okrem vibračných stavov taktiež aj rotačné stavy reprezentované nízko-ležiacimi 2+, 4+, 6+... hladinami, ktoré narastajú úmerne $J(J+1)$.

Rotačné + jednočasticové stavy

Rotačný stav, môže byť „vybudovaný“ na jednotlivých jednočasticových stavoch. V prípade nepárno-párnych jadier sa tým výrazne zvyšuje hustota hladín a komplikuje sa analýza identifikovaných prechodov.